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TOPICAL REVIEW

New particles in strong fields?

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Abstract. The recent search for new particles in the MeV mass range is reviewed. The most relevant experiments, most notably the GSI positron experiments, are described and their results are combined to restrict the properties of the hypothetical particles. The theoretical analyses of these and a large number of other experiments are presented in detail. Finally some special candidates for new particles such as the axion and localised complex vacuum excitations are discussed. I discuss a number of ideas and suggestions which merit further investigation. With respect to the GSI experiments it is concluded that it is very unlikely that the observed e^+–e^- coincidences indicate the existence of new particles.

1. Introduction

The general consensus among most elementary particle physicists is that all phenomena up to a typical energy scale of 100 GeV can be described by the standard model, and that qualitative new physics can only be expected above this energy scale. This comfortable feeling of certainty, however, has recently been disturbed by some unexplained experimental results. The best studied are the monoenergetic electron–positron pairs produced in a special class of heavy ion reactions, which have been observed at GSI (Gesellschaft für Schwerionenforschung, Darmstadt, West Germany). These data were interpreted as evidence that new light particles with mass between 1 and 2 MeV might exist, an hypothesis which motivated an enormous amount of theoretical and experimental work.

As several other unexplained experimental results have been claimed over the past years which might be accounted for by a new light particle, several attempts were made to relate some of these particle hypotheses to the GSI data.

These manifold efforts so far have not produced any convincing argument for the existence of new light particles. For the GSI experiments, in particular, very strong arguments could be formulated which all but rule out any particle interpretation. At the same time this work has, however, raised many new questions and led to the development of a variety of new ideas. Furthermore the arguments developed are so general that they will have to be taken into account by every future hypothesis postulating the existence of new light particles.

In the course of this whole discussion it has become clear that the mass region between about 1 MeV and 100 MeV is very hard to investigate experimentally with high accuracy. The only high precision theory we really have is QED, which is associated with the electron mass scale and therefore is rather insensitive to masses

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substantially larger than 1 MeV. Hadronic reactions and decays which are sensitive to larger masses are generally rather poorly known, so that side channels with a small branching ratio would usually pass unnoticed. The only exceptions are decays of long-lived hadrons like the kaons, the J/ψ, or the Υ, which, however, give stringent bounds only for point particles.

In this report we shall try to summarise the results obtained so far and the ideas which still have to be investigated. In doing so we shall focus on the particle interpretation of the GSI peaks, but most of the experiments, calculations and speculations presented are of more general relevance. Ideas rejected as an explanation for the positron peaks could still be employed in a different context.

This review covers only the situation for light bosons; for fermions a similar comprehensive analysis has not yet been done.

At this point we want to recall briefly the original motivation of the GSI experiments. For a point charge with $Z>137$ the binding energy of electrons exceeds twice their rest mass (see figure 1). If such nuclei were fully ionised electron–positron pairs would be created spontaneously. The electron would be bound and the positron emitted. Such nuclei do not exist in nature, but Greiner and his collaborators have suggested over many years that signals for this process could be observed in heavy ion collisions in which nuclear systems with a charge number up to 188 are produced for a short moment. This field of research and the calculations necessary for definite predictions are very involved. An excellent overview can be found in [1]. The most interesting aspect of this work for our review is that sharp positron lines were predicted [2, 3] under the assumption that the colliding nuclei would stick together for an extremely long time (about 100 times longer than one would usually expect).

The positron spectra subsequently measured indeed showed such lines, the systematics of which were, however, incompatible with creation by spontaneous vacuum decay. We therefore proposed in 1984 [4] that such lines could be produced

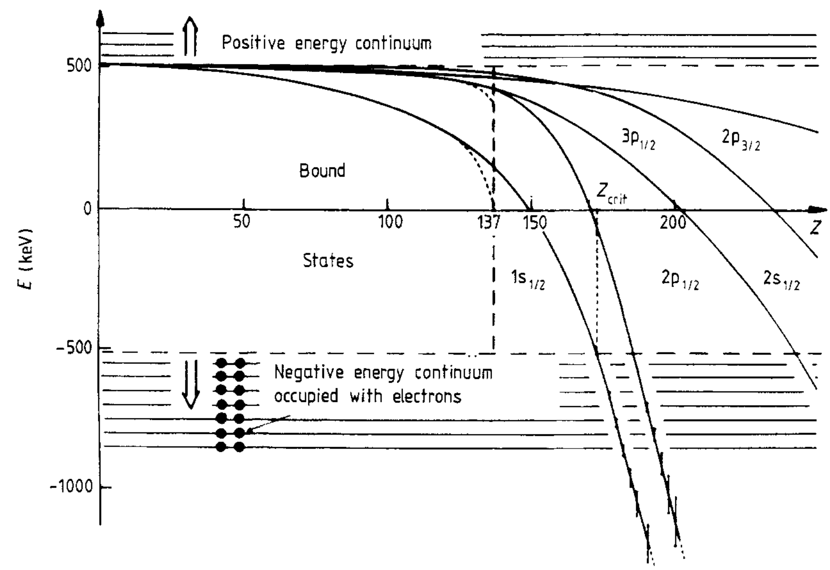

Figure 1. Atomic binding energies as a function of nuclear charge.

by the two-body decay of some new particle. In 1985 [5] we argued that various high precision experiments provide very stringent constraints for such a particle hypothesis. Since then these bounds have been substantially improved up to a point where the particle hypothesis can be nearly ruled out.

This review is organised as follows. In § 2 the crucial experiments are discussed for a particle interpretation of the positron lines. In this section complementary experiments are also reviewed and it is shown in particular that beam-dump experiments rule out that a new point particle might cause the GSI coincidences.

Different ideas on the nature of the hypothetical particles (which we shall call X throughout) have been discussed. The simplest possibility is to assume that they are point particles, described by a new quantum field, and that their interaction with other particles is just given by a simple Lagrangian. One is then able to calculate all the interesting processes and can derive bounds for the relevant parameters.

One result of these investigations, which are reported in § 3, is that such an elementary particle could never be produced in sufficient numbers in heavy ion collisions to explain the observed electron–positron coincidences. The most fascinating candidate for a new light particle is the axion. Many articles have been written about the possibility that the GSI experiments might have discovered the axion. Quite a few of them had the tendency to ignore many of the experimental facts and concentrate on the special features of their models. Due to this fact the different axion models developed in this context (discussed in § 5) are still interesting even if the GSI data turn out to be unreliable and should therefore be regarded as independent speculations. They are alternatives to the standard axion models and might become relevant in a different context. The whole discussion about the axion showed that the tests of the standard model at low energies had been somewhat incomplete, and possibly still are.

Section 6 will address speculations about new particles with some internal structure. These speculations can be divided in two groups. Those in the first group postulate unusual complex states within the standard model, like localised regions with a modified QED-, Higgs- or QCD-vacuum or strongly bound magnetic resonances. Those in the second postulate completely new interactions. As most of the models which have been proposed are based on non-perturbative effects, which are very hard to calculate, many of these hypotheses seem at first impossible to rule out completely. Nevertheless many counter-arguments have been found, so that all of these speculations appear unpromising, at least as explanations for the GSI data.

In this review we shall emphasise those speculations and propositions which seem to be promising for future research. Extensive reviews on the GSI experiments, their motivation and interpretation and on the axion discussion can be found in [6] and [7].

2. Experimental results

The main problem of any hypothesis postulating new light particles is to avoid contradictions with the multitude of experiments in atomic, nuclear and elementary particle physics which are in perfect agreement with the standard model. We are not going to discuss in detail all the different experiments which lead to interesting bounds for new light particles. We shall only cite their results whenever they are needed. Instead we concentrate on those experiments which suggest the existence of new particles. The best studied, and for a while the most promising, are the GSI experiments and those directly connected with them. In § 2.2 we discuss some other experiments from different areas of physics.

2.1. *The GSI positron experiments*

Originally the positron experiments at GSI were planned to test characteristic effects of the very strong electromagnetic fields produced in heavy ion reactions. If the combined charge number of the two nuclei is approximately 160 the number of positrons produced by the time-varying electromagnetic fields (at projectile energies close to the Coulomb threshold) is comparable to those produced by nuclear transitions. For still larger charges these so called 'dynamical positrons' are dominant. They have a broad smooth spectrum, the width of their distribution being just the inverse of the typical nuclear reaction times of the order of 10^{-21} s, namely 600 keV. They are perfectly understood: theory and experiment agree without any adjustable parameters within 10% or at most 20% and this high degree of knowledge can in turn be used to obtain precise measurements for nuclear reaction times as a function of bombarding energy.

For the problem we are interested in these dynamical positrons are treated as background. By 1980 [8] narrow positron lines sticking out of this background had been reported for heavy ion collisions just at or slightly below the Coulomb threshold, although with only marginal statistics. Such lines had been predicted as a signal of spontaneous e^+–e^- production in very strong electric fields [2, 3]. Many apparatus and systematic problems had to be overcome before the existence of these lines was firmly established independently by the ORANGE and EPOS groups. The third group involved in positron experiments at GSI, namely the TORI group, concentrated on heavy ion collisions with much higher energies for which no lines were observed.

The EPOS and ORANGE spectrometers are shown in figures 2 and 3. For details of the apparatus we refer to the articles in [9]. Fortunately, both set-ups are completely different, which excluded nearly all purely apparatus-related origins of the positron lines from the beginning.

In the ORANGE spectrometer (figure 2) the projectiles pass first through a hole in the positron counter before reaching the target. The positrons are then focused by magnetic fields onto specific sections of the detector, which gives two independent energy measurements: the signal of the detector itself and the position on the counter. The nuclear fragments are detected in coincidence with the positrons. In the past two years a second spectrometer was added (to the right in figure 2) which now serves to detect electrons also in coincidence with the positrons and nuclear fragments.

In contrast to the geometry used by the ORANGE spectrometer the electrons and positrons are transported perpendicular to the beam axis in the EPOS experiment (figure 3). Furthermore the positrons are identified by their 511 keV gamma lines, ruling out any error in the particle identification.

In the early experiments the electrons were not detected. These data are referred to as 'single spectra' as distinguished from the 'coincidence spectra' to be discussed shortly. In comparing all the single spectra which have been published or reported at conferences one is faced with a rather confusing situation. Most of these spectra show structures, but the precise energies of these lines differ from system to system and the latest spectra usually show several lines. It is unfortunate that for only a few systems do several comparable spectra exist and very often the data available have such poor statistics that their interpretation is ambiguous. Comparable experiments agree about as often as they disagree.

The main reason for these and other difficulties is, however, probably just statistics. To get a good spectrum one would typically need a month of beam time. One would like to study several different systems at several different bombarding energies, and one would prefer every experiment to be performed independently by different groups. As the Unilac accelerator at GSI is used by many groups such a programme is clearly out of the question. Thus the experimentalists have to put up with often rather unsatisfactory compromises. For example, to obtain the largest counting rate possible the beam has to be very intense which creates all kinds of target problems. Often a target exposed to the beam will be substantially corroded and such damaged targets do not show lines. The bombarding energy becomes ill defined in such a target because of wildly varying energy losses. If the cross section for the coincident pairs were strongly energy dependent this could completely blur the results. In practice the targets are checked by proton-backscattering to single out the 'good' targets. This process, however, clearly introduces some arbitrariness one would prefer to avoid.

Another example of the problems caused by the poor statistics are the cuts introduced in the early experiments to increase the signal to background ratio. These cuts are made with respect to the kinematics of the nuclear fragments which are measured in coincidence with the positrons. By choosing them judiciously one can extract a peak which might otherwise be lost in the background. This procedure can be justified by the reasonable assumption that the monoenergetic positrons come only from a very specific class of nuclear reactions, but again it introduces some unwanted uncertainties. In particular it is possible that the additional lines reported in the later

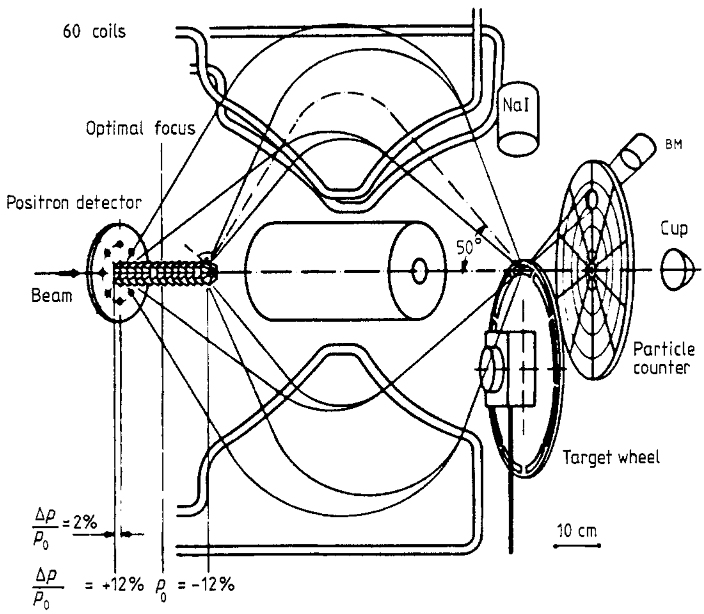

Figure 2. The ORANGE spectrometer in its early form [12]. Recently a second similar spectrometer was added to the right to detect coincident electrons and positrons. Reproduced with permission from GSI.

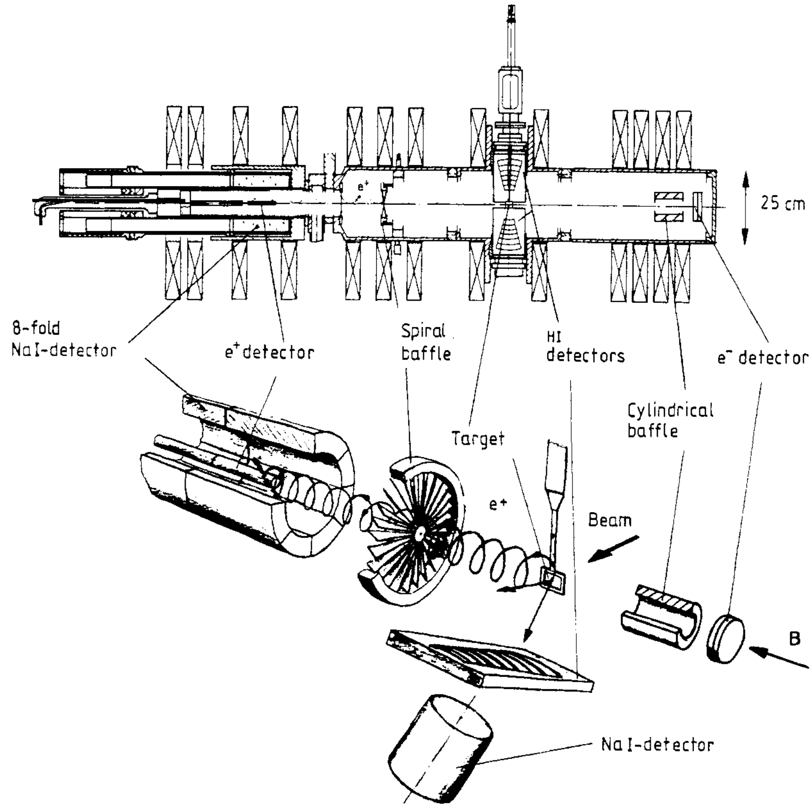

Figure 3. The EPOS experiment [9] in its configuration used to detect the coincident e^+–e^- pairs. Reproduced with permission from Plenum Press.

experiments were overlooked in the earlier experiments just because of this procedure. Luckily these cuts are not necessary any more in the newest experiments.

We do not want to discuss all the experimental data accumulated in the past eight years. Instead we shall just show a few typical results.

Figure 4 shows the collection of single positron spectra published by the EPOS group in 1985 [10] which first started speculation that the positron lines could be caused by the two-body decay of a new particle. The different spectra shown are obtained for a wide range of total charge numbers. In each spectrum a broad background due to dynamical positron production and nuclear conversion underlies a clear peak at about 300 keV kinetic energy. The most striking feature of these spectra is their extreme similarity, which seems to rule out any atomic origin of these structures such as spontaneous positron production [6]. (Most crucial is the Th + Ta spectrum which is undercritical and thus should not show any spontaneous positron production.) The line energies for the different systems are at the 2σ level, all compatible with 330 keV.

The second example is a recent compilation of spectra by the ORANGE group obtained for several light systems (figure 5) [11]. Two line structures at about 243 and 314 keV are visible. Comparing figures 4 and 5 it is, however, obvious that the cross section of the peak relative to the background is much smaller in the ORANGE data than in the EPOS data. Furthermore, a detailed analysis shows that the energy of the higher lying lines of the ORANGE spectra (which this group claims to be only uncertain by 5 keV) is not compatible with the 375±10 keV reported by the EPOS group for Th + Ta. This difference can only partially be explained by the Doppler effect in connection with a suitable angular distribution of the positrons. The remaining differences are still extremely disturbing. Similar contradictions are reported for the U + U system.

While the differences in the reported line positions are alarming, those in the cross sections could possibly just reflect the differences in the way the actual experiments are carried out. The EPOS group takes the position that the observed lines are very strongly dependent on the exact beam energy and the quality of the target, and therefore they closely monitor their spectra online to fine-tune the beam energy and exchange targets if they seem to be deteriorating. In addition the spectra in figure 4 were obtained by optimising cuts on the nuclear scattering angles. If there is a clear well defined signal this is obviously a legitimate procedure to increase the peak to background ratio. However, it has led to the criticism that some of the structures might be artificially cultivated.

In contrast the ORANGE group takes the position that the experiments should be done under constant conditions, which in fact means averaging over all random changes caused by small shifts in the beam energy and possible target effects. In principle this procedure should guarantee the reproducibility of their results (up to purely statistical fluctuations which are, however, crucial for the small signal to background ratios in most ORANGE spectra). This is reasonable if the line structures are *not* strongly dependent on the beam energy.

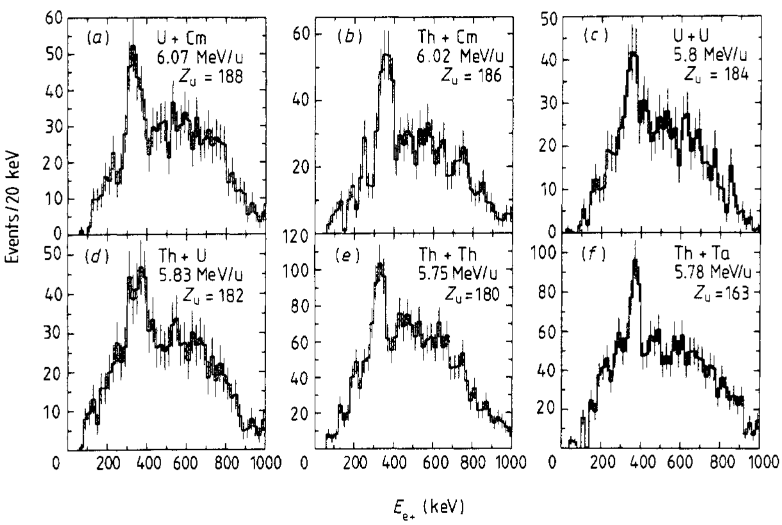

Figure 4. Collection of positron spectra published by the EPOS group in 1985 [10]. Reproduced with permission from *Phys. Rev. Lett.*

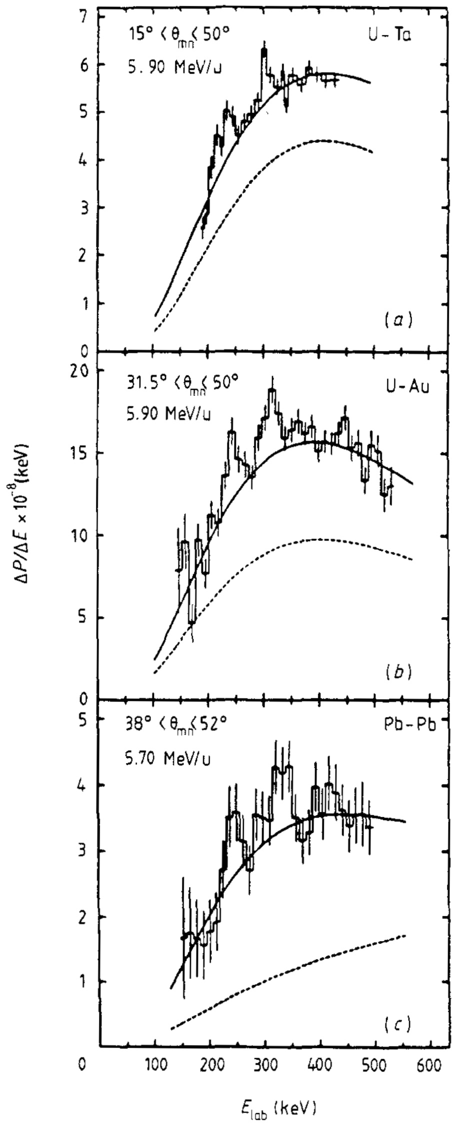

Figure 5. Spectra for three low Z systems published by the ORANGE group in 1987 [11]. Full curve, total calculated background; broken curve, nuclear background alone. Reproduced with permission from *Z. Phys.*

These differences in general approach to experiments are also reflected in the fact that in the EPOS spectra only 'counts/channel' are plotted whereas the ORANGE group claims to measure real cross sections.

If one puts all the lines ever published into one plot one finds that there is hardly any energy between 220 and 430 KeV which has not been claimed at some time and in some system as the position of a line.

For the coincidence lines the situation seems to be somewhat clearer, which could, however, also be due to the fact that there is not yet so much data available. Several lines in the sum-energy spectrum of coincident electron–positron pairs have been observed. In each case the electron spectra and positron spectra also show well defined lines. The situation for the difference spectra, however, which are of absolute crucial importance for the interpretation, is unclear.

Finally, recent experiments testing the angular correlation between the coincident electrons and positrons led to conflicting results.

Let us start our discussion of these data with figure 6 which, when they became known, were quite sensational. In an earlier paper we had speculated about the possibility of some charged object decaying into a positron–neutrino pair [4]. Later we concluded that this idea would not work [5]. The EPOS group searched instead for a corresponding electron–positron decay of some neutral object.

In the upper part of the figure the number of e^+–e^- pairs are plotted according to their electron and positron kinetic energy. If one just adds up all the positrons and all

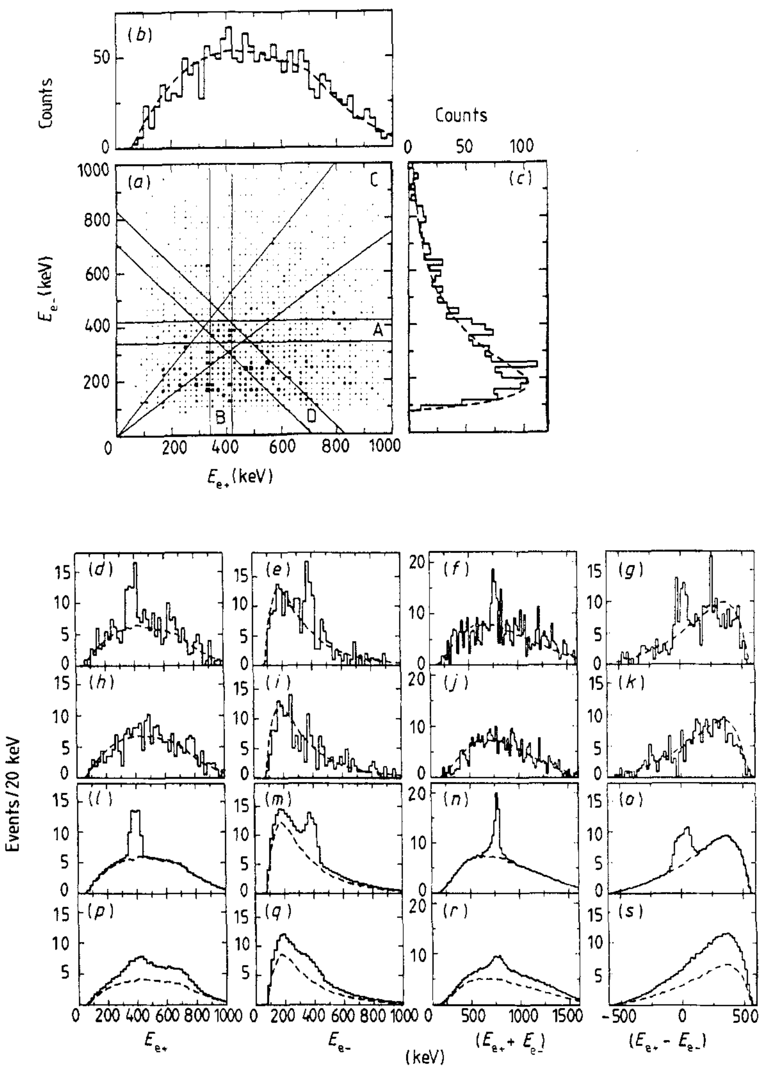

Figure 6. The data from the first EPOS coincidence experiment [105]. Reproduced with permission from *Phys. Rev. Lett.*

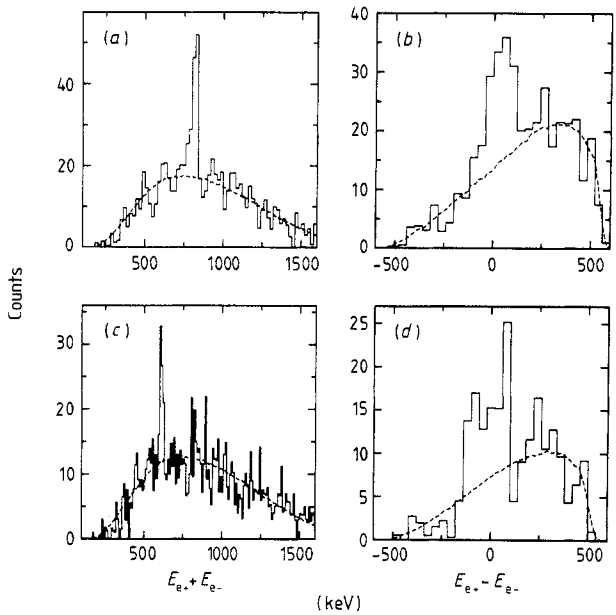

Figure 7. The data from the second EPOS coincidence experiment [9]. Reproduced with permission from Plenum Press.

the electrons the spectra at the right side and the top are obtained, showing no significant line structure. If, however, only the data in the regions A to D are plotted against their respective energy variable a clear peak becomes visible not only in the electron and positron spectrum, but also in the sum-energy and difference-energy spectrum: figures *d–g*. In figures *h–k* the data from the regions adjacent to region A to D are plotted in a similar way showing no indication for a line. This implies that there is an excess of counts in the overlap region of A to D. The figures *l–o* show the result of a Monte Carlo simulation for the spectra *d–g* assuming that the decay of a particle with 1.7 MeV mass would be responsible for these additional counts. Figure *p–s* show a similar simulation assuming that the coincident electron position pairs are due to nuclear pair conversion. It is obvious that figures *l–o* fit the experimental data perfectly. The system studied in this case was uranium on thorium.

The only problem with these data at that time seemed to be that the positron energy of these coincidences is about 380 keV, significantly larger than that observed in the single spectra (figure 4).

This experiment was repeated hoping to improve the statistics so much that the decay kinematics suggested by figure 6 would be firmly established. Surprisingly the line which was so clear in figure 6 is not visible in any of the later published data, although the EPOS group claims that a faint remnant of it can be extracted. Instead two new sum-energy lines showed up, one at 1.8 MeV and one at 1.6 MeV. The two sum- and difference-energy spectra are shown in figure 7. While the 1.8 MeV line is very prominent the one at 1.6 MeV is only so clearly distinguishable if the positron is required to arrive at the detector about 3 ns after the usual arrival time for dynamical positrons. The significance of this time delay is still unclear. The most convincing explanation is that the e^+–e^- pairs responsible for the 1.6 MeV line show some definite angular distribution and that the delay time is just caused by the relatively long way the positrons have to travel through the magnetic field.

In the past year nearly the same results were obtained for the undercritical system U + Ta (one sumline at 620 KeV total kinetic energy associated with delayed positrons and one line at 1.805 keV in the 'prompt' spectrum). A 'critical' system is one in which the combined charge is larger than the value needed to bind an electron with more than twice its rest mass. Depending on the radius of the nuclear system and the amount of screening this value is typically between $Z = 170$ and $Z = 180$ (for a point charge it is 137) [1].

The 750 keV line was also clearly detected in this experiment. In addition the EPOS collaboration split their positron and electron detectors into several parts, allowing them to get some crude angular correlation. They could basically only decide whether the electron or positron was emitted in the, respectively, forward or backward hemisphere. Both the 1.8 MeV and 1.6 MeV lines are visible only if e^+ and e^- are emitted in different hemispheres, which would support the particle interpretation, but the angular resolution is so bad that some atomic process with a suitable angular correlation might also explain this result. The 1.77 MeV line, however, is seen dominantly if e^+ and e^- are emitted in the same hemisphere. The difference-energy spectra have very poor statistics. For the 1.8 MeV and 1.6 MeV lines they at least do not clearly rule out a two-body decay, whereas for the 1.77 MeV line it seems to show at best a much too broad structure. Finally the U + Ta system is the only one of those studied which shows some structure in the gamma spectrum at the matching energy, which further supports the suspicion that the 750 keV line might be caused by nuclear conversion. As this whole analysis is still preliminary all this information has to be taken with care.

Meanwhile the ORANGE group has also performed their first coincidence experiment using the U + U system. Their experiment has a better, though still very coarse, angular resolution. The detectors have a hexagonal shape in the plane orthogonal to the beam. This allows the relative angle of electron and positron within this plane to be determined with an uncertainty of about 60°. The preliminary data [12], see figure 8, seem to indicate that there are several sum-energy lines which appear only in the 180° bin. Again the statistics are very poor implying a much smaller cross section than that suggested by the EPOS data (see, however, the discussion above). The line at about 1.84 MeV is statistically probably significant, whereas the lower-energy structures should be interpreted with care.

What does all this mean for the hypothesis that a new particle has been discovered in these experiments? If one postulates that there is only one new particle which one might try to identify with the 1.8 MeV state then the energetically lower lines would have to be associated with either bound states of this X particle or completely independent processes. The second alternative is not very attractive because if conventional mechanisms existed to explain these states it would be very surprising if they could not also account for the 1.8 MeV state. The most discussed of such mechanisms is nuclear conversion. All experimental and theoretical studies made so far seem to rule it out, except perhaps for the U + Ta system (even there the expected contribution calculated from the observed gamma intensity is much smaller than the observed areas of the lines), but as these nuclear systems are very unusual it cannot be excluded altogether that these studies were insufficient.

The first possibility, namely that exotic bound states of the X particles might exist, has to be investigated theoretically in more detail. *A priori* it is, however, exceedingly unlikely that in the decay of an X bound to, for example, one of the nuclei the electrons and positrons produced should still have nearly the same energy.

An alternative is the postulate of several new particles, in the most extreme case one for every line observed. This is a still less attractive posibility as it introduces more and more rather arbitrary parameters (unless one could construct some model which would give the correct mass spectrum). Even if one were willing to do this the explanation of all the experimental details which do not support the particle interpretation would certainly require a multitude of *ad hoc* assumptions.

A sensational step such as the postulation of a new light particle, and thus some new physics, at the MeV level can only be justified by absolutely compelling data. The experimental foundation provided by the GSI experiments is at best very shaky and does not justify such a claim. It should, however, also be noted that the experiments do not provide any real clue as to which other process might create the coincidence lines.

Experimentally the heavy ion experiments would have to be upgraded in such a way that the invariant mass of the electron–positron pair can be measured precisely. Only if a clear peak in the invariant mass spectrum were observed could the discovery of a new particle be claimed. Plans in this direction do exist (e.g. at Argonne).

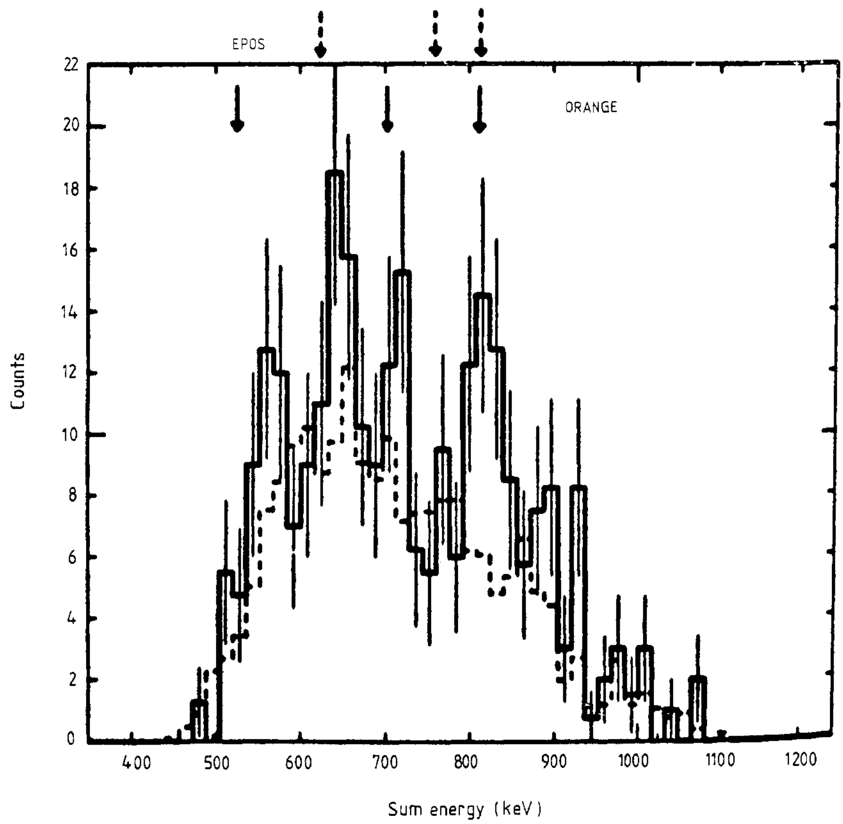

Figure 8. Preliminary data from the first ORANGE coincidence experiment [12] (full curve, 180°; broken curve, 0°). Reproduced with permission from GSI.

In the following discussion it will become clear that theoretical investigations provide additional and even stronger arguments against the particle hypothesis.

2.2. Complementary particle search experiments

We have pointed out in the last section that the main problem of the GSI experiments is acquiring sufficient statistics. It would therefore be extremely important to find some other experimental way of studying the processes involved, and many attempts in this direction have been made. While most of these experiments were designed to find a signature for the hypothetical X particle the first of them tried to check whether the explanation might be a completely different one. Erb *et al* [13] started from the hypothesis that the line structures could be produced by some complicated exotic solid state effect. This hypothesis sounds very unrealistic as the typical energy scale of solid state physics is eV and not 100 keV, but all other hypotheses are also unrealistic.

In their experiments they substituted the heavy ion collision as a most ineffective positron source by a conventional one, namely as ^{68}Ge–^{68}Ga sample. The positrons from this source hit a thorium target and coincident electron–positron pairs were detected in two 'mini-oranges' (scaled down versions of the ORANGE spectrometer). Surprisingly, the first run of this experiment indeed showed a small, but just significant, structure at positron energies of 340 keV. It turned out, however, that this structure was the Compton edge of photons scattering off electrons in the positron detector. In a second run the experiment was improved by requiring the observation of 511 keV annihilation quanta for the positron identification and indeed the structure disappeared.

Meanwhile several other groups had set out to repeat this experiment [14–17]. Two of them [15, 16] have found a structure which, however, is statistically hardly significant and would be in contradiction with the other three experiments. It is generally accepted by now that there are not any exotic effects in the positron spectra of the size observed in the GSI experiments.

A different approach was chosen by Meyerhof *et al* [18, 19]. They basically repeated the heavy ion experiments but searched for correlated gamma pairs instead of e^+–e^- pairs. As anything which can decay into electron–positron pairs can also decay into photons, such correlations should exist at some level. If it is suppressed by a factor α^2 or α^3 then the search for these correlations is futile, but if the gamma decay were of similar importance to the e^+–e^- decay then it would be easier to detect. The limitation of their experiment is that it was only sufficiently sensitive to look for two-gamma coincidences. The decay of a spin-one particle, which can only decay into three gammas, would therefore not be observed. This experiment did not find any gamma–gamma coincidences at the energies of the sumlines observed at GSI, but in the second run they claimed that a structure at 1062 ± 1 keV showed exactly the right width and Doppler corrections as expected for a two-photon decay of a particle. As this energy is just above the threshold energy for e^+–e^- production it seemed reasonable that the hypothetical particle would decay to a larger fraction into gammas than its more massive partners associated with the GSI coincidences. The corresponding cross section would have been comparable to the e^+–e^- cross sections deduced for the GSI lines. A third run with an improved apparatus, collecting about eight times more statistics, did not confirm these results. Meanwhile, the 1062 keV line has been identified as due to nuclear deexcitation.

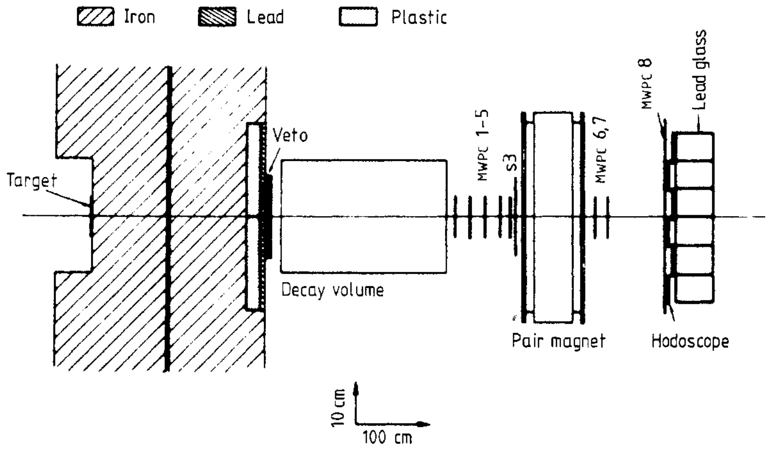

Figure 9. The beam-dump experiment of Konaka *et al* [22]. Reproduced with permission from *Phys. Rev. Lett.*

In contrast to heavy ion experiments, with their typically large backgrounds, experiments with electrons or electrons and positrons are much easier to interpret. Furthermore, some of the production mechanisms of X particles in heavy ion collisions are hard to calculate reliably and exotic processes might be overlooked.

As the hypothetical X particle is supposed to decay into e^+ and e^- it must also be possible to produce it in e^+–e^- scattering or as bremsstrahlung emitted by electrons [20]. Both have been tried by several different groups. Perhaps the most important set of experiments for all searches for light weakly interacting particles are the beam-dump experiments. A proton or electron beam of high energy is stopped in a high-Z beam dump. Any light particle coupling to electrons or nucleons should then be copiously produced in the resulting cascade. As all hypothetical new light particles can only interact rather weakly with normal matter as they would otherwise have been noticed in other experiments (this point is discussed extensively and quantified in the rest of this review) they should be able to leave the beam-dump and thus be identified in downstream detectors. This technique is primarily limited by the production cross section and the lifetime of the particle one is searching for. The mean free path (i.e. the lifetime times the gamma factor times the speed of light) has to lie somewhere in the range of 0.3 m to perhaps 3 km. If it is much shorter any interesting object produced in the beam-dump will also decay in it, if it is much larger the probability of it decaying in the detector becomes too small.

For the case of the X particle the situation is especially simple as both the production and the decay are assumed to be governed by the coupling to the electron (if other production mechanisms are also effective this would only strengthen the bounds). The non-observation of any signal can therefore be converted into clear constraints for the X–electron coupling constant α_{Xe} (for its definition see §3). By great experimental effort and by using different beams the various beam-dump experiments [21–25] covered the whole range from 10^{-7} to 10^{-14} in this parameter. Figure 9 shows a typical experimental set-up namely that of Konaka *et al* [22]. In all of these experiments one looked for unaccounted e^+–e^- pairs appearing downstream of the beam-dump. None of the various experiments found any indication for such pairs, which lead to the bounds given in the table 1 (for $M_X = 1.8$ MeV)

There are two caveats to these conclusions. The first is that, if the X–nucleon cross section were very large, the X particles could be absorbed in the beam-dump and/or the shielding. Riordan *et al* [23] have analysed this possibility thoroughly and found that the X nucleon cross section would have to be of the order of 50 mb to get a substantial effect. As this is of the order of the pion-nucleon cross section such a strong coupling is impossible. If it existed the nucleon–nucleon interaction would be dominated by long range X-exchange.

The second possibility is that the X particle could be extended. As the momentum transfers involved are of the order of GeV even for an X radius of only a few fm a form factor could suppress the X bremsstrahlung substantially. We shall observe in § 6 that this is, however, not the case. The bounds from the beam-dump experiments prove to be crucial even for extended objects. To invalidate them X radii of at least 100 fm would have to be postulated. (Such large radii have also been suggested by e.g. Schramm *et al* [26].)

We want to stress that beam-dump experiments are probably the most powerful and most generally applicable way to search for light weakly interacting particles.

As very large X radii could invalidate the constraints obtained from the beam-dump experiments the e^+–e^- scattering experiments are important to close this last loophole. These experiments search for resonances in the Bhabha cross section in the range between 1.5 and 2 MeV total energy. As the momentum transfers involved are exactly the same as in the GSI experiments the coupling constant has to be the same whether or not it includes a form factor. The problem with these experiments is that the X interaction has to compete with the normal electromagnetic one, which is stronger by many orders of magnitude. Therefore a very good energy resolution is needed to extract the resonance. A detailed calculation ([27] and § 3) shows that the ratio of the resonance cross section to the ordinary Bhabha cross section is of the order $\alpha_{\mathrm{Xe}} m/\alpha^2 \Delta E$, where ΔE is the total energy resolution which for the fixed target experiments presently under way is dominated by the Fermi motion of the electrons in the target. This ratio implies that to be sensitive to the whole range of α_{Xe} down to 10^{-11} one needs an energy resolution of the order of 0.1 eV. In the experiments done so far [28–33] using a positron beam and a fixed low-Z target ΔE was about 20 keV so that the interesting range of α_{Xe} could not be tested.

Surprisingly a clear signal was claimed by the Stuttgart group [32]. A consecutive run with increased sensitivity showed, however, that at least this original claim was wrong. The South African group [28] published a claim based on very poor statistics

Table 1.

Group	Beam	Result
Riordan *et al* [23] SLAC	9 GeV e^-	with X from electron bremsstrahlung $\alpha_{\mathrm{Xe}} > 10^{-7}$ or $\alpha_{\mathrm{Xe}} < 10^{-12}$
Konaka *et al* [22] KEK	2.5 GeV e^-	$\alpha_{\mathrm{Xe}} > 10^{-8}$ or $\alpha_{\mathrm{Xe}} < 10^{-14}$
Brown *et al* [24] Fermilab	800 GeV p	$\alpha_{\mathrm{Xe}} > 10^{-7}$ or $\alpha_{\mathrm{Xe}} < 10^{-10}$
Davier *et al* [25] Orsay	1.5 GeV e^-	$\alpha_{\mathrm{Xe}} > 10^{-8}$ or $\alpha_{\mathrm{Xe}} < 10^{-11}$
Bechis *et al* [21] Bethesda	45 MeV e^-	$\alpha_{\mathrm{Xe}} > 10^{-10}$ or $\alpha_{\mathrm{Xe}} < 10^{-13}$

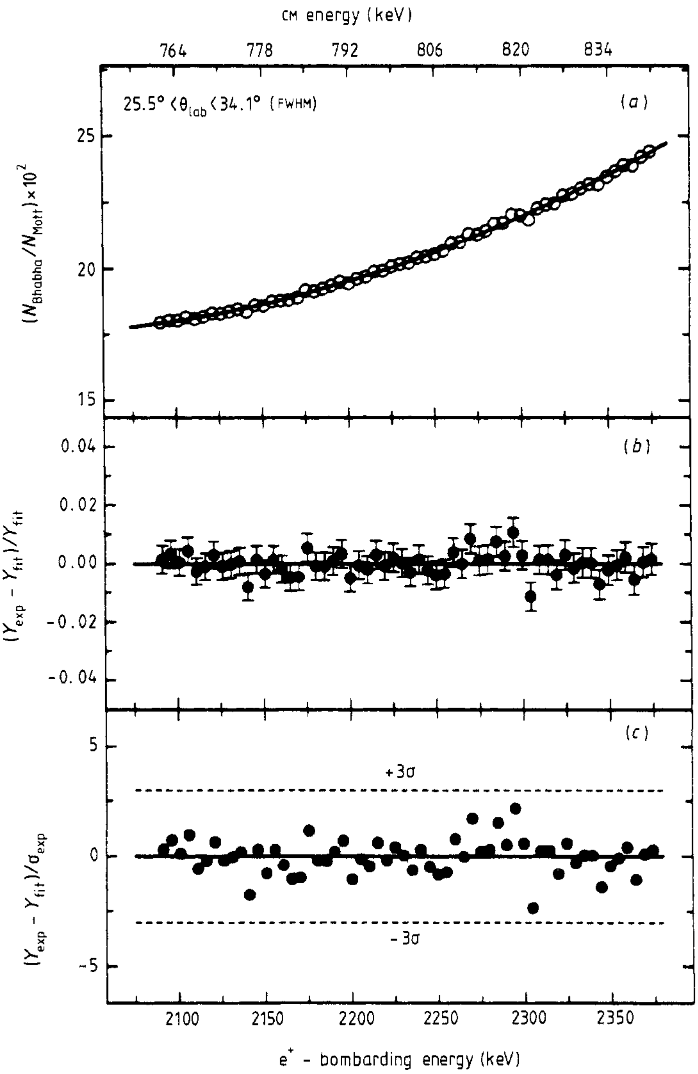

Figure 10. The Bhabha experiment of Tsertos *et al* [33]. Reproduced with permission from *Phys. Lett. B*.

which was obviously unjustified. The best spectrum available so far was obtained by Tsertos *et al* [33] and is shown in figure 10. It clearly does not show any indiction for a resonance. To obtain the better energy resolution necessary for a really crucial experiment future set-ups will have to use different techniques. Plans do exist for a scheme which uses a magnetic bottle to create a positron target and a very high quality electron beam (T Cowan, private communication). The most fascinating possibility would be a minute low-energy e^+–e^- synchroton. Although this idea was raised by several people its feasibility seems to be very questionable.

Finally another set of experiments directly connected to the GSI experiments have been searching for X production in nuclear conversion [34–38]. The idea behind these experiments was that nuclear processes might be responsible for the X production. Most of them, in fact, focused on the possibility that the X might be the axion and therefore searched only for pseudoscalars.

The expected cross sections depend on the X–proton, X–neutron and the X–electron coupling (as the X particles can only be identified by their decay into electron–positron pairs). Earlier experiments [39–41] which have not been looking for a short-lived particle like the hypothetical X are not sensitive to lifetimes less than 10^{-11} s because of shielding placed between the target and the detector. The new experiments of this type were therefore constructed in such a manner as to be sensitive to shorter lifetimes.

As two different coupling constants are involved, one for the proton and one for the neutron, two different decays have to be analysed for every J^{π} assignment of the X particle to obtain a complete set of constraints. So far this has been only done for the pseudoscalar case.

The proton and neutron coupling is parametrised in terms of an isoscalar and isovector coupling constant

$$L = \bar{\psi}\gamma_5(g^{(0)} + g^{(1)}\tau_3)\psi X. \tag{1}$$

The transitions studied are:

M1 transition of ^{10}B with 3.59 MeV [35]
M1 transition of ^{13}C with 3.68 MeV [42]
M1 transition of ^{14}N with 9.17 MeV [34, 38]
E0 transition of ^{16}O with 6.05 MeV [38]
M1 and E2 transitions of ^{8}Be [38].

No experiment has found any statistically significant signal for X conversion which can be translated into bounds which are typically of the order [38]

$$g^{(0)} < 1.6 \times 10^{-2} \qquad g^{(1)} < 2.0 \times 10^{-2}. \tag{2}$$

In conclusion all the different complementary experiments searching for the X particle have found no indication for its existence yet do provide stringent bounds on its properties.

2.3. *Some other particle speculations*

In recent years quite a few other anomalies have been claimed which could be explained by the existence of a new light particle. These include anomalies in positronium decay and level spacing [42, 43], in the near threshold production of e^+–e^- pairs [44] and an apparent discrepancy for the anomalous magnetic moment of the electron (see §3). The latter has now been resolved and the former are not convincing enough to be taken seriously.

In this section we shall discuss therefore only one such anomaly, namely the muon-rich cascades caused by particles coming from Cygnus-X. Very energetic particle showers have been observed which have the periodicity of Cygnus-X and come from a direction which is compatible with its position. These showers are produced by particles with an energy between 10^6 and 10^7 GeV hitting the Earth's atmosphere [45–47]. The still unanswered question is what these particles are. They have to be neutral as otherwise they would have been deflected by the interstellar magnetic fields. They also have to be rather light (i.e. lighter than 10 GeV) as otherwise the phase information would be washed out and they finally have to fulfil the constraint that their lifetime multiplied by the gamma factor has to be large enough for them to reach the Earth:

$$\tau \frac{10^6 \mathrm{GeV}}{M} \geqslant 1.3 \times 10^{12}\,\mathrm{s}. \tag{3}$$

The only known particle which fulfils all these requirements is the photon. The problem with this identification is, however, that the photon showers should contain only about 10% of the muons observed.

Many speculations concerning these exotic events have been published [48–50]. We do not want to discuss them here. The most probable explanation seems to be that the present understanding of the formation of air showers is insufficient and that also the muon-rich showers are created by photons.

Instead we want to concentrate on the suggestion that this particle might be a low-mass (i.e. with a mass below the e^+–e^- threshold) component of the same class of states to which the X particles belong, and we shall demonstrate that this is not possible.

If the Cygnus-X3 candidate had anything to do with the X particle this would imply that its coupling to the electron and its mass should be comparable. Then, for example, a pseudoscalar particle, the lifetime is limited from above by the two-photon decay proceeding through the formation of a virtual e^+–e^- pair. This lifetime is well known from studying $\pi \rightarrow \gamma\gamma$ to be [51]

$$\tau \approx \frac{4\pi^2 m_e^2}{\alpha^2 \alpha_{Xe} M^3}. \tag{4}$$

From the GSI experiments we know that $\alpha_{Xe} > 10^{-11}$ and we chose $M = 1$ MeV which implies

$$\tau\gamma < 10^5\,\mathrm{s} \tag{5}$$

a much too short decay time. For a scalar one gets a similar value.

We conclude that, whatever the final interpretation of the Cygnus-X3 events might turn out to be, it is exceedingly unlikely that any connection to the GSI data can be justified.

From the discussion in this and the last section two points should have become clear.

(1) There are a large variety of problems in many different fields where the bounds derived in the context of the GSI data can be used to constrain or rule out different hypotheses.

(2) Most exotic anomalies claimed so far have turned out to have conventional though not always simple explanations. Therefore rock-solid evidence is needed before any far-reaching hypothesis for their explanation should be taken seriously.

3. Bounds for pointlike particles

The main problem of any hypothesis postulating the existence of new light particles is the fact that they have not been observed in all the other experiments done in the past 50 years. There are basically two ways in which this can be explained. The first possibility is obviously that the coupling constants of this new particle are chosen small enough (how small will be specified in this section). The second possibility is that the hypothetical particle is provided with such special properties that it can only be produced under most unusual conditions, e.g. very strong electromagnetic fields in the case of the GSI particle hypothesis, and very strong and extended magnetic fields in the case of the Cygnus-X3 particle hypothesis. Such possibilities are discussed in §6 for the GSI coincidence data.

Whereas the second possibility requires very specific assumptions for each individual problem one is looking at, the constraints on the coupling constants in the absence of any exotic phenomena are universally valid.

One of the most precisely measured (if not the most precisely measured) quantity in physics is the anomalous magnetic moment of the electron. As any particle coupling to the electron contributes to the vertex correction it also contributes to the anomalous magnetic moment.

The simplest graph contributing to g-2 [52] is shown in figure 11. If one assumes the hypothetical X particle to be elementary and thus be described by a renormalisable field theory its coupling to the electron is uniquely determined by its spin and parity:

$$L(x) = g_{\mathrm{Xe}}\bar{e}(x)\Gamma\varphi e(x) \tag{6}$$

with

$$\begin{array}{lllll} J^{\pi} = & 0^{+} & 0^{-} & 1^{+} & 1^{-} \\ \Gamma\varphi = & \varphi & i\gamma_5\varphi & \gamma_\mu\varphi^\mu & \gamma_\mu\gamma_5\varphi_\mu . \end{array}$$

The g-2 experiment, however, not only provides constraints for the direct coupling but also for much more fanciful interactions. Figure 12 shows an example. In this case a bound for the product of g_{Xe} and some effective X–two-photon coupling is obtained.

The theoretical and experimental values for the anomalous magnetic moment of the electron as well as for the muon are [53–56].

$$a_{\mathrm{e}}(\text{theory}) = 1\,159\,652\,460\,(128)\,(43) \times 10^{-12} \tag{7}$$

$$a_\mu(\text{theory}) = 11\,659\,202\,(20) \times 10^{-10}. \tag{8}$$

The first uncertainty for the electron is due to the uncertainty of the fine-structure constant. The second error as well as the uncertainty for the muon are due to higher,

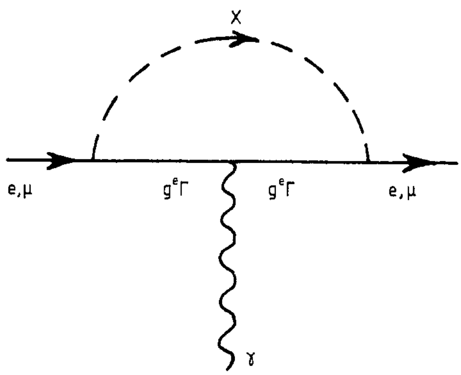

Figure 11. The lowest-order contribution of an X particle to the anomalous magnetic moments of electron and muon.

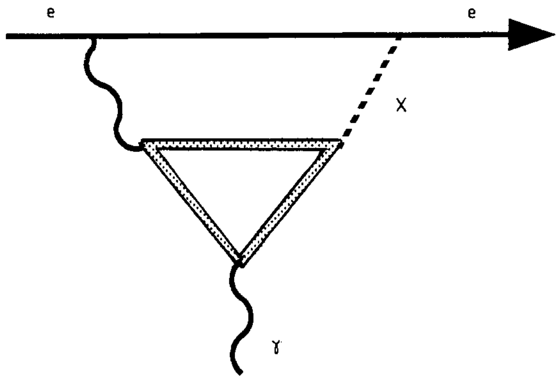

Figure 12. More complex contributifon of an X particle to the anomalous magnetic moments of electron and muon. The shaded line indicates an arbitrary charged particle.

not yet calculated, graphs:

$$a_e(\exp) = 1\,159\,652\,193\,(10) \times 10^{-12} \tag{9}$$

$$a_\mu(\exp) = 11\,659\,230\,(84) \times 10^{-10}. \tag{10}$$

While the overall agreement is excellent the 2σ discrepancy for a_e provoked a host of theoretical speculations (a very exotic example can be found in [57], where it was interpreted as evidence for the fractal dimension of spacetime). Luckily the origin of this discrepancy was found recently (a fine description of the situation can be found in [58]). To avoid a circular argument the fine structure has to be determined by some other means than g-2, namely by a combination of experiments on the integral quantised Hall effect and an AC Josephson effect. Both the experiments from which α was derived used the NBS standard resistance, the value of which was then eliminated by combining their results. It turned out, however, that the NBS standard resistance changed slightly over the years. As the two experiments were done at different times this drift lead to a slight error in α and thus to a slightly incorrect prediction for a_e. If a correction is made assuming a drift linear in time the resulting prediction for a_e agrees perfectly with the experimental value allowing only for a discrepancy

$$\Delta a_e \leqslant 10^{-10}. \tag{11}$$

The contribution from figure 11 can be calculated easily. Its contribution to the vertex function is

$$\Lambda_\mu(p', p) = \pm \frac{\alpha_{Xe}}{4\pi} \int dz_1\, dz_2\, dz_3\, \delta(1 - z_1 - z_2 - z_3) \tag{12}$$

$$\frac{\Gamma_X[p'\gamma(1-z_2) - p\gamma z_3 + m]\gamma_\mu[p\gamma(1-z_3) - p'\gamma z_2 + m]\Gamma_X}{m^2(1-z_1)^2 + m_X^2 z_1 - q^2 z_2 z_3}$$

with the plus sign for spin-zero particles and the minus sign for spin-one particles. Γ_x stands for the appropriate gamma matrices. The momentum of the incoming (outgoing) Dirac particle is p (p'). The relevant quantity is the term multiplying $\sigma_{\mu\nu}q^\nu$ which is finite. The calculation is therefore unproblematic, leading to the result

$$\Delta a_e = \frac{\alpha_{Xe}}{2\pi} K_i\left(\frac{m_X}{m}\right) \tag{13}$$

with the functions K_i plotted in figure 13.

$$
\begin{aligned}
K_S &= \tfrac{3}{2} - \rho + \tfrac{1}{2}\rho(\rho-3)\ln\rho - (\rho^2 - 5\rho + 4)F(\rho) \\
K_P &= -\tfrac{1}{2} - \rho + \tfrac{1}{2}\rho(\rho-1)\ln\rho - (\rho^2 - 3\rho)F(\rho) \\
K_V &= 1 - 2\rho + \rho(\rho-2)\ln\rho - 2(\rho^2 - 4\rho + 2)F(\rho) \\
K_A &= 9 - 2\rho + (\rho^2 - 6\rho + 4)\ln\rho - 2(\rho^2 - 8\rho + 14)F(\rho)
\end{aligned}
\tag{14}
$$

$$
F(\rho) = \sqrt{\frac{\rho}{\rho-4}}\,\mathrm{Arth}\left(\sqrt{\frac{\rho-4}{\rho}}\right) \qquad \text{if } \rho > 4
$$

$$
F(\rho) = \sqrt{\frac{\rho}{4-\rho}}\,\tan^{-1}\left(\sqrt{\frac{4-\rho}{\rho}}\right) \qquad \text{if } \rho < 4 \tag{15}
$$

$$
\rho = (m_X/m_e)^2. \tag{16}
$$

We are thus able to conclude for the GSI problem for which m_X is about 1.7 MeV ($\alpha_{Xe} = g_{Xe}^2/4\pi$):

$$
\begin{aligned}
&\alpha_{Xe}^S < 3\times10^{-9} &\qquad &\alpha_{Xe}^P < 4\times10^{-9} \\
&\alpha_{Xe}^V < 1\times10^{-8} &\qquad &\alpha_{Xe}^A < 2\times10^{-9}.
\end{aligned}
\tag{17}
$$

Clearly the bounds for a_e become insensitive if the mass of the hypothetical particle becomes much larger than the electron mass. Equation 16 and figure 13 show that for $\rho \gg 1$ the contribution is proportional to $1/\rho$:

$$
\begin{aligned}
&K_S \to \frac{\ln\rho}{\rho} &\qquad &K_P \to -\frac{\ln\rho}{\rho} \\
&K_V \to \frac{2}{3\rho} &\qquad &K_P \to -\frac{10}{3\rho}.
\end{aligned}
\tag{18}
$$

Thus even for particles with GeV mass the coupling to the electron has to fulfil $a_{Xe} < 10^{-2}$.

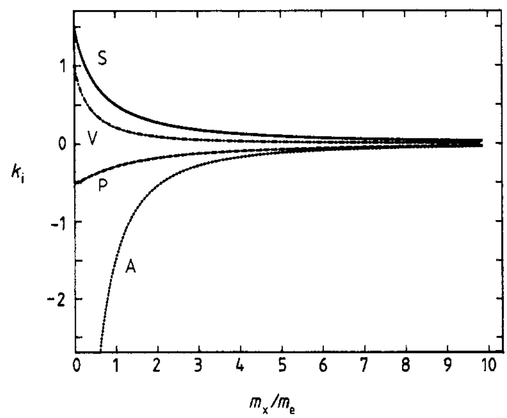

Figure 13. The functions K_i parametrising the contribution of the X particle to the anomalous magnetic moment.

For the muon the uncertainty in the anomalous magnetic moment is larger by a factor 100, $\Delta a_\mu < 1\times10^{-8}$ but also the muon mass is much larger. For a 1.7 MeV particle the resulting bounds are slightly less stringent than in the electron case:

$$\alpha^{\mathrm{S}}_{\mathrm{X}\mu} < 4\times10^{-8} \qquad \alpha^{\mathrm{P}}_{\mathrm{X}\mu} < 1\times10^{-7}$$

$$\alpha^{\mathrm{V}}_{\mathrm{X}\mu} < 7\times10^{-8} \qquad \alpha^{\mathrm{A}}_{\mathrm{X}\mu} < 3\times10^{-9} \tag{19}$$

but for more massive X particles they are substantially better. The combination of both bounds is therefore interesting for all scenarios postulating new particles with mass below 1 GeV.

The graph in figure 12 cannot be calculated in general terms. As the loop moments have to be integrated over, the effective X–two-photon coupling has to be known for all momentum transfers. This requires, however, the knowledge of with which particles the broken lines have to be identified. In general the situation can become very complicated. (If the X is, for example, assumed to be an axial vector and the particle corresponding to the shaded line to be a fermion the triangle graph can have an anomaly.)

A problem with the bounds derived from the anomalous magnetic moments is that the contributions from different particles can cancel (see figure 13). Although it will normally require a very unattractive fine tuning of the various couplings this possibility must be kept in mind.

The hyperfine splitting of positronium is another precision experiment which would be influenced by any new particle coupling to the electron. Figure 14 shows the relevant processes. The energy shift for a given positronium state is just

$$\Delta E = \frac{g^2_{\mathrm{Xe}}}{m^2_{\mathrm{X}} - 4m^2_{\mathrm{e}}} A - \frac{g^2_{\mathrm{Xe}}}{m^2_{\mathrm{X}}} B \tag{20}$$

$$A = (\bar{v}\Gamma_{\mathrm{X}} u)(\bar{u}\Gamma_{\mathrm{X}} v) \qquad B = (\bar{v}\Gamma_{\mathrm{X}} v)(\bar{u}\Gamma_{\mathrm{X}} u). \tag{21}$$

Depending on the spin and parity of the X particle this gives

$$J^\pi = 0^+ : A = 0 \qquad B = |\varphi(0)|^2$$

$$J^\pi = 0^- : A = \frac{1 - \langle \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \rangle}{2} |\varphi(0)|^2 \qquad B = 0$$

$$J^\pi = 1^+ : A = \frac{1 - \langle \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \rangle}{2} |\varphi(0)|^2 \qquad B = \langle \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \rangle |\varphi(0)|^2$$

$$J^\pi = 1^- : A = -\frac{3 + \langle \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \rangle}{2} |\varphi(0)|^2 \qquad B = |\varphi(0)|^2 \tag{22}$$

with $\varphi(0)$ being the non-relativistic wavefunction at the origin. For angular momentum zero and principal quantum number n one has

$$|\varphi(0)|^2 = \alpha^3 m^3 / 8\pi n^3. \tag{23}$$

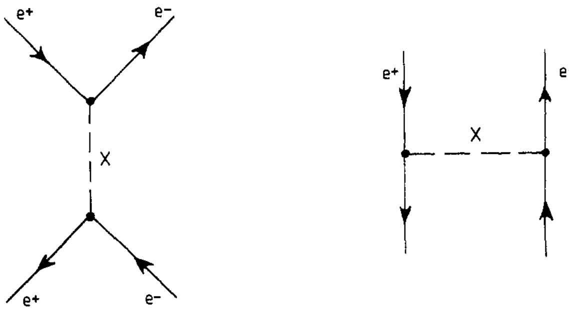

Figure 14. Graphs contributing to the hyperfine splitting of positronium.

The experimental and theoretical uncertainty of the energy difference between the 1^3S_1 and 1^1S_0 level are less than 10 MHz [59–61]. For $M_X = 1.7$ MeV this leads to the bounds

$$\alpha^{P}_{Xe} < 10^{-6} \qquad \alpha^{V}_{Xe} < 10^{-6} \qquad \alpha^{A}_{Xe} < 7 \times 10^{-7}. \tag{24}$$

These bounds have to be taken as absolute as no cancellation can occur in this case and a finite size of the X particle does not change anything (unless this size becomes comparable to the positronium radius). To get a similar bound for a scalar particle precise calculations and measurements for the 1^3S_1 and 2^3S_1 energy difference would be needed.

The contribution to the level spacings in positronium as to the anomalous magnetic moments are proportional to $1/M_X^2$. Thus unless some substantial cancellation occurs or the internal structure of the X particle changes drastically its contribution to the anomalous magnetic moments the bounds from positronium are always less accurate.

For heavier X particles in the GeV range the same arguments could be repeated for the muonium or charmonium system, the properties of which are naturally far less precisely known.

As we have mentioned in § 2.3 there are some unsettled discrepancies for the level spacing between the 1^3S_1 and 2^3S_1 state [42] and for the decay constants of ortho-positronium [43]. We believe that these are most probably just due to insufficient theoretical calculations.

Low-energy electron–positron scattering is very similar in character to the lowest-energy bound-state interaction in positronium. The relevant graphs are actually identical (see figure 14) differing only in the choice of the incoming and outgoing electron and positron states. Compared with the positronium experiments electron–positron scattering has the advantage that the X resonance can be isolated and thus the signal to background ratio enhanced to a degree that depends only on the energy resolution achieved. In practice it has led to bounds two orders of magnitude better (see § 2.3) so far and there is still some hope for further improvements.

We shall give here a short review of the rather complete discussion in reference [27]. The analysis starts with the simplest case, assuming free electrons and positrons. The calculation is then analogous to that for usual Bhabha scattering [62]. If $p_{1\mu}$, $p_{2\mu}$

are the momenta of the incoming positron and electron and $p'_{1\mu}$, $p'_{2\mu}$ those of the outgoing ones the cross section takes the form

$$\mathrm{d}\sigma_i = \frac{m^4}{\sqrt{p_1 \cdot p_2 - m^4}} \frac{\mathrm{d}^3 \boldsymbol{p}'_1}{(2\pi)^3 E'_1} \frac{\mathrm{d}^3 \boldsymbol{p}'_2}{(2\pi)^3 E'_2} (2\pi)^4 \delta^4(p'_{1\mu} + p'_{2\mu} - p_{1\mu} - p_{2\mu}) |M_{\mathrm{fi}}^{(\mathrm{i})}|^2 \tag{25}$$

with

$$M_{\mathrm{fi}}^{(\mathrm{i})} = (g_{\mathrm{Xe}}^{\mathrm{i}})^2 \bar{u}(p'_2, s'_2) \Gamma_{\mathrm{X}} v(p'_1, s'_1) \frac{1}{(p_1 + p_2)^2 - (m_{\mathrm{x}} - i\Gamma/2)^2} \bar{v}(p_1, s_1) \Gamma_{\mathrm{X}} u(p_2, s_2)$$

$$\text{if } i = \mathrm{P}, \mathrm{S} \tag{26}$$

$$M_{\mathrm{fi}}^{(\mathrm{i})} = (g_{\mathrm{Xe}}^{\mathrm{i}})^2 \bar{u}(p'_2, s'_2) \Gamma_{\mathrm{X}}^{\mu} v(p'_1, s'_1) \frac{-g_{\mu\nu} + (p_1 + p_2)_\mu (p_1 + p_2)_\nu / m_{\mathrm{X}}^2}{(p_1 + p_2)^2 - (m_{\mathrm{x}} - i\Gamma/2)^2}$$

$$\times \bar{v}(p_1, s_1) \Gamma_{\mathrm{X}}^{\nu} u(p_2, s_2) \qquad \text{if } i = \mathrm{V}, \mathrm{A}. \tag{27}$$

Γ is the decay width of the X particle and m the electron mass. If the e^+–e^- decay is the dominant decay channel Γ is related to $\alpha_{\mathrm{Xe}}^{\mathrm{i}}$ by

$$\Gamma \approx \alpha_{\mathrm{Xe}}^{\mathrm{i}} m G_{\mathrm{i}}(\rho) \tag{28}$$

with $\rho = m_{\mathrm{X}}/m$ and

$$G_{\mathrm{S}} = \frac{(\rho^2 - 4)^{3/2}}{2\rho^2} \qquad G_{\mathrm{P}} = \frac{(\rho^2 - 4)^{1/2}}{2}$$

$$G_{\mathrm{V}} = \frac{(\rho^2 - 4)^{1/2}(\rho^2 + 2)}{3\rho^2} \qquad G_{\mathrm{A}} = \frac{(\rho^2 - 4)^{3/2}}{3\rho^2}. \tag{29}$$

After averaging over the initial spin directions and summing over the final spins equation (25) gives

$$\frac{\mathrm{d}\bar{\sigma}_{\mathrm{i}}}{\mathrm{d}\Omega'_1} = \frac{\alpha_{\mathrm{i}}^2}{m^2} \frac{\cos\theta'_1}{(1 - b^2 \cos^2\theta'_1)^2} \frac{m^2}{(E + m)^2} \frac{m^2}{(E - E_{\mathrm{R}})^2 + (\rho\Gamma_{\mathrm{i}}/2)^2} S_{\mathrm{i}}(s, t) \tag{30}$$

with

$$s = (p_1 + p_2)^2 \qquad t = (p'_1 - p_1)^2$$

$$E_{\mathrm{R}} = \frac{m_{\mathrm{x}}^2 - 2m^2 - \Gamma^2/4}{2m} \qquad b^2 = \frac{E - m}{E + m}$$

and

$$S_{\mathrm{S}}(s, t) = \frac{(s - 4m^2)^2}{4m^4}$$

$$S_{\mathrm{P}}(s, t) = \frac{s^2}{4m^4}$$

$$S_{\mathrm{V}}(s, t) = \frac{s^2/2 + 4m^2 + (s - 4m^2)t + t^2}{m^4}$$

$$S_{\mathrm{A}}(s, t) = \frac{s^2/2 - 4m^2 s + 12m^4 - 8m^2 s/\rho^2 + 4s^2/\rho^4 + (s - 4m^2)t + t^2}{m^4}. \tag{31}$$

The resonance width $\rho\Gamma_i$ is at most a few meV and thus always much smaller than the energy spread of the incoming electrons and positrons ΔE. Therefore the experiments cannot measure the actual resonance cross section (which shows the interference with the usual Bhabha scattering) but only an energy average of it (which has a bell-shaped form). The treatment of the energy spread of the positrons in the fixed target experiments discussed in §2.2 is relatively easy:

$$\langle\sigma_i(E)\rangle = \frac{\Delta E}{2\pi}\int \mathrm{d}E' \frac{\sigma(E')}{(E'-E)^2+(\Delta E/2)^2} \qquad \Delta E \gg \rho\Gamma_i. \tag{32}$$

The resulting differential cross sections are shown in figure 15 in the laboratory frame. The units are α^2/m^2 and $E_{\mathrm{kin}} = 2.262$ MeV, corresponding to a centre-of-mass (CM) energy of 1.832 MeV which is assumed to be the X mass. The thick curve shows ordinary Bhabha scattering while the other lines (which still have to be multiplied by $(\Gamma^{i}_{e^+e^-}/\Gamma^{i})(\alpha_{\mathrm{Xe}} m/\Delta E))$ show the contribution from the resonance scattering for the different multipolarities: 0^+ (full curve), 0^- (broken curve), 1^+ (dotted curve), and 1^- (chain curve).

In the experiments performed so far the energy spread was, however, dominated by the momentum spread of the bound electrons which is harder to treat precisely. The matrix element $M^{(i)}_{\mathrm{fi}}$ in equation (25) depends then on the electron wavefunctions in momentum space $\psi_n(\boldsymbol{q})$. For the spin-averaged cross section one gets [27]

$$\bar{\sigma}(E) = 2\pi^2(m/p^2)g^2_{\mathrm{Xe}}\Theta(E+E_n-m_X)\int_{|p-p_X|}^{p+p_X} \mathrm{d}_{qq}F_{i,n}(\boldsymbol{q},\boldsymbol{p}) \tag{33}$$

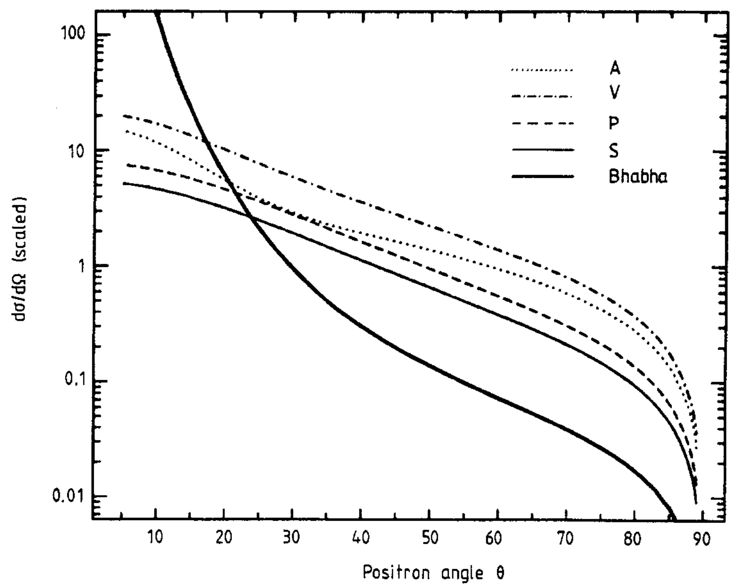

Figure 15. Differential Bhabha cross section (for explanation see text).

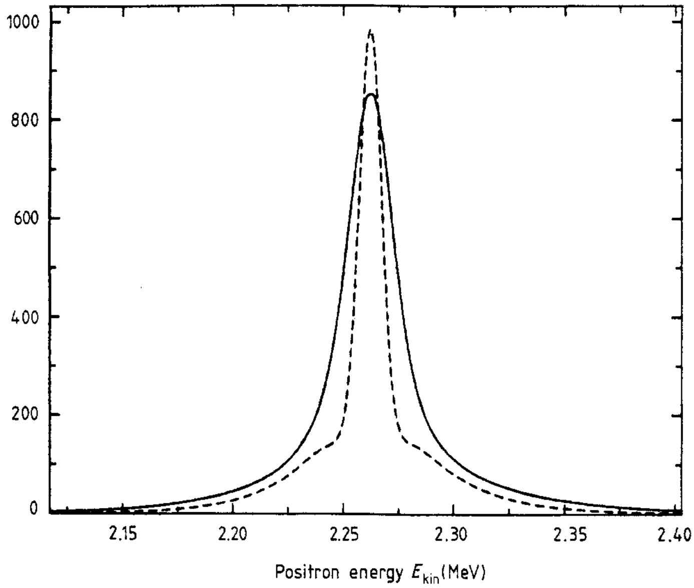

Figure 16. The X production cross section σ_X in units of α_X-m^2 for e^+ scattering off screened bound electrons for a pseudoscalar X particle. The X mass was assumed to be 1.832 MeV.

with

$$F_{i,n} = \tfrac{1}{2} \sum_s |\bar{v}(p, s)\Gamma_i \psi_n(\mathbf{q})|^2 \qquad \text{for } i = \mathrm{S, P}$$

$$F_{i,n} = \tfrac{1}{2} \sum_\lambda \sum_s |\bar{v}(p, s)\Gamma_i^\mu \psi_n(q)\varepsilon_\mu^{(\lambda)}(p_X)|^2 \qquad \text{for } i = \mathrm{V, A}. \tag{34}$$

This cross section has then to be summed over all bound states n as well as the delocalised molecular states. The result of such a calculation is shown in figure 16.

Low-energy Bhabha scattering is a tool which can be used for all searches for new light particles coupling to electrons. The major problem is that the Bhabha experiments are only very sensitive for extremely high-energy resolution, which implies that one has to know the energy of the particle one is looking for with high accuracy to make the experiments possible.

The great advantage of Bhabha scattering compared with the g-2 bounds is that again a cancellation of contributions from different particles is not possible (at least not for all polarisations) and that the internal structure of the X particle should not have any importance. The X particle is off the mass shell by typically only 30 keV and thus only structures at the scale of 10^4 fm can be resolved.

Another way to search for new light particles is to investigate the long range part of the nucleon–nucleon potential. This has already been done several years ago by Barbieri and Ericson [63] who analysed neutron–nucleus scattering. An additional light scalar particle would lead to an additional Yukawa potential. For a sufficiently light X they obtained the bound $\alpha^{\mathrm{s}}_{\mathrm{Xn}} < 3 \times 10^{-10}$ which is even stronger than that derived from g-2 for the X–electron coupling. Unfortunately such bounds can only be obtained for the scalar and axial-vectorial case. (An axial-vectorial particle couples to the spin-density.) Pseudoscalar or vector particles couple to the static nucleus only by relativistic effects. For these cases an analysis of definite partial waves in high-energy nucleon–nucleon scattering might lead to some reasonable bounds. So far such calculations have not been performed.

As it became clear that one is not able to find a consistent scenario to explain the GSI data by the decay of a new light particle using the couplings of equation (6) one turned next to effective phenomenological couplings. The most interesting of these are direct X–photon couplings as they could lead to new effects related to the strong electromagnetic fields generated in heavy-ion collisions:

$$\begin{aligned}
J^{\pi}=0^{+}&: L=\tfrac{1}{2}g^{\mathrm{S}}_{\mathrm{X}\gamma}F_{\mu\nu}F^{\mu\nu}\varphi=g^{\mathrm{S}}_{\mathrm{X}\gamma}(E^{2}-B^{2})\varphi\\
J^{\pi}=0^{-}&: L=\tfrac{1}{4}g^{\mathrm{P}}_{\mathrm{X}\gamma}F_{\mu\nu}\tilde{F}^{\mu\nu}\varphi=g^{\mathrm{P}}_{\mathrm{X}\gamma}\boldsymbol{E}\cdot\boldsymbol{B}\varphi\\
J^{\pi}=1^{+}&: L=\tfrac{1}{4}g^{\mathrm{A}}_{\mathrm{X}\gamma}F_{\mu\nu}\tilde{F}^{\mu\nu}F_{\lambda\rho}\Phi^{\lambda\rho}+\tfrac{1}{4}g'^{\mathrm{A}}_{\mathrm{X}\gamma}F_{\mu\nu}F^{\mu\nu}\tilde{F}_{\lambda\rho}\Phi^{\lambda\rho}\\
J^{\pi}=1^{-}&: L=\tfrac{1}{4}g^{\mathrm{V}}_{\mathrm{X}\gamma}F_{\mu\nu}F^{\mu\nu}F_{\lambda\rho}\Phi^{\lambda\rho}+\tfrac{1}{4}g'^{\mathrm{V}}_{\mathrm{X}\gamma}\tilde{F}_{\mu\nu}F^{\mu\nu}\tilde{F}_{\lambda\rho}\Phi^{\lambda\rho}\\
&\qquad+\tfrac{1}{2}g''^{\mathrm{V}}_{\mathrm{X}\gamma}\partial_{\lambda}F_{\mu\nu}\partial^{\lambda}\Phi^{\mu\nu}
\end{aligned} \tag{35}$$

with

$$\Phi^{\mu\nu}=\partial^{\mu}\varphi^{\nu}-\partial^{\nu}\varphi^{\mu}. \tag{36}$$

Again known experimental data can be used to limit the relevant coupling constants. So far this has been done for nuclear pair conversion [64] and Delbrück scattering [65]. The graphs leading to additional contributions in these two cases are shown in figures 17 and 18. The relevant matrix elements for Delbrück scattering are calculated in analogy to [66]. Figure 18(a) gives for the pseudoscalar case

$$\langle\boldsymbol{k}'|M|\boldsymbol{k}\rangle=\frac{\omega(g^{\mathrm{P}}_{\mathrm{X}\gamma})^{2}}{2}\int \mathrm{d}^{3}q\int \mathrm{d}^{3}q'\,\delta^{3}(\boldsymbol{k}-\boldsymbol{k}'-\boldsymbol{q}-\boldsymbol{q}')\,\frac{\boldsymbol{E}(q)\cdot\boldsymbol{\varepsilon}_{\perp}\boldsymbol{E}(q')\cdot\boldsymbol{\varepsilon}'_{\perp}}{(\boldsymbol{k}+\boldsymbol{q})^{2}+m^{2}_{\mathrm{X}}}. \tag{37}$$

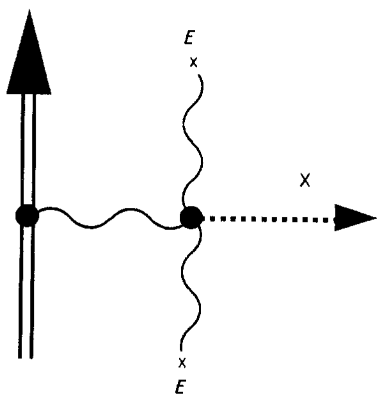

Figure 17. Lowest-order contribution of an X particle to nuclear pair conversion.

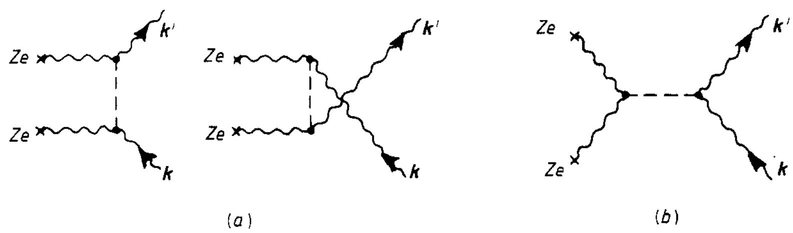

Figure 18. Lowest order contribution of an X particle to Delbrück scattering.

$\boldsymbol{k}$ and $\boldsymbol{k}'$ are the momenta of the incoming and outgoing photons and $\varepsilon_\perp$, $\varepsilon'_\perp$ are the respective transverse polarisation vectors. $\boldsymbol{E}(\boldsymbol{q})$ is the Coulomb field in momentum space. The cross section is simply

$$\mathrm{d}\sigma/\mathrm{d}\Omega = \tfrac{1}{2} \sum_{\varepsilon, \varepsilon'} (2\pi\omega)^2 |\langle \boldsymbol{k}'|M|\boldsymbol{k}\rangle|^2. \tag{38}$$

We calculated the contributions to the Delbrück cross section measured in a specific experiment [67] using uranium as a target for a scalar and pseudoscalar particle. The bounds we thus obtained are

$$\alpha^{\mathrm{S}}_{\mathrm{X}\gamma} < 5 \times 10^{-4}\,\mathrm{MeV}^{-1} \qquad \alpha^{\mathrm{P}}_{\mathrm{X}\gamma} < 2 \times 10^{-5}\,\mathrm{MeV}^{-1}. \tag{39}$$

To obtain similar bounds for the vector and axial vector case we also analysed pair conversion in heavy nuclei [64]. In the electric field of the nucleus an emitted photon can convert into a X which later on decays into an electron–positron pair. Thus the X contributes to the pair conversion coefficient and bounds for its couplings can be derived comparing the measured and calculated values. For the current coupling to the $\Phi_\mu(x)$ field we chose

$$J^\mu_{\mathrm{X}} = 2g^{\mathrm{j}\mu}\partial^0(E^{\mathrm{j}}E^{\mathrm{k}}\Phi^{0\mathrm{k}}) + 2g^{0\mu}\partial^{\mathrm{j}}(E^{\mathrm{j}}E^{\mathrm{k}}\Phi^{0\mathrm{k}}) + \partial_\nu(E^2\Phi^{\nu\mu}) \tag{40}$$

and calculated the pair conversion coefficient for an E3 transition in lead with 2.615 MeV. Comparing our result with the experimental value [68] and the QED prediction [69] we got

$$g^{\mathrm{V}}_{\mathrm{X}\gamma} < 3 \times 10^{-9}\,\mathrm{MeV}^{-4}\sqrt{W_{\mathrm{e}}}. \tag{41}$$

(W_{e} is the probability that an X particle decays into an electron–positron pair.)

The constraints thus obtained for the X–gamma coupling are, however, very sensitive to the size of the X particle. As the basic scale for both processes is the nuclear radius any object with a radius much larger than 10 fm could only lead to a much reduced correction.

Similar, though less stringent, bounds can be derived from an analysis of K_α energies and the Lamb shift [20].

4. Consequences for the GSI data

All the bounds just discussed severely limit the possibilities to explain the production of substantial amounts of X particles in heavy ion collisions. The cross section of the positron lines observed in heavy ion collisions of roughly 100 μb implies that one X would have to be produced in about every 10^4th collision. Thus X production would not be an exceedingly rare process. On the other hand we have seen that the coupling of the X particle to electrons and nucleons is much smaller than the electromagnetic one.

This problem is further aggravated by the fact that the X particles have to decay nearly at rest in the centre of mass system. Otherwise the positron lines produced by its two-body decay would be smeared out in the laboratory system. To solve this problem it was noticed [20] that a rather long X lifetime of at least 10^{-8} s could guarantee that the fast X particles decay primarily outside of the sensitive regions of the detectors. This in turn could lead to sharp positron lines even for a broad X spectrum in the CM system. This possibility has meanwhile been ruled out by experiments at GSI which proved that the X lifetime has to be shorter than 10^{-9} s.

Another possibility discussed intensively for some time was that the X mass could be just slightly larger than twice the electron mass and that it would be produced with some definite energies corresponding to the observed sum-energies of the electron–positron coincidences. In this case electron and positron would be emitted in nearly the same direction, which was ruled out by the latest coincidence experiments (see figure 8).

Thus a very efficient and selective production mechanism, producing predominantly low-momentum X particles, would be needed to explain the observed cross sections.

This conclusion is confirmed by calculations which have been done for conventional production mechanisms such as electron bremsstrahlung, nuclear bremsstrahlung, production due to a coupling to collective nuclear modes, and production due to the direct X–photon coupling. None of these mechanisms gets even near to the required production rates. For details see [6].

We do not want to review these calculations in detail. When they were first done they ruled out a large number of X particle scenarios and were therefore very important. In the meantime, however, the same conclusions can be reached more stringently along the lines we present in this review, such that a detailed presentation is no longer necessary.

It became clear that the bounds from g-2 together with the negative results from the beam-dump experiments require any particle candidate for the GSI coincidences (if it is a scalar, pseudoscalar, vector or axial vector) to be much larger than the nuclei involved. Thus the calculations done so far for point particles grossly overestimate the production rates, typically by a factor $(R_{\text{nuclei}}/R_X)^n$ with some fairly large n.

If one corrects the results obtained for the processes mentioned above by inserting such a factor with $R_X > 100$ fm the total cross sections are too small by typically at least ten orders of magnitude.

Thus the bounds derived in §3 require an exotic, probably non-perturbative, production mechanism, such as a condensation process or a Landau–Zehner type of adiabatic conversion process, for the postulated X particle. No model should be taken seriously unless it includes such a process. Papers which just try to fit the 'observed' line spectrum with some more or less *ad hoc* model without addressing the problem of the production cross section are of little use.

5. The search for the axion

So far we have discussed different experimental findings which might suggest the existence of new light particles and constraints on their coupling constants. In the following two sections we analyse how such particles might fit into the standard model. As the standard model is so extremely successful any hypothesis which not only postulates the existence of new particles but also that there is new physics at low energy which cannot be incorporated into the standard model is doubly improbable. If, however, the hypothetical particles were incorporated in a natural way into the standard model they would become much more acceptable (this is why we tried to find such a possibility from the very beginning [4, 5]). The fact that this is not possible (or at least has not been possible so far) is one of the reasons why the particle interpretation of the GSI data is so very unattractive.

Again a distinction has to be made between point particles and extended composite objects. Within the standard model the latter would be identified with some complex bound state consisting of electrons and photons, or gluons and quarks, or some complex excitations associated with the Higgs vacuum or a mixture of all of these. We shall discuss this possibility in the next section.

Point particles have been conclusively ruled out by the beam-dump experiments as an explanation for the GSI data (see §§6.1 and 2.2). In view of other particle speculations and as a part of the general search for new light particles we nevertheless want to discuss whether and how they can be incorporated into the standard model. For vectorial and axial-vectorial particles this is not possible as they would have to be identified with new gauge fields corresponding to an enlarged gauge group and thus a new interaction. Scalars and pseudoscalars can, however, be incorporated into the Higgs sector. The Higgs sector is still very poorly understood. It is questionable whether it has any physical reality at all, or whether it is only an effective parametrisation of some more complicated processes. Furthermore the Higgs sector does not only allow for the introduction of new particles, but there are even some theoretical arguments why at least one such new pseudoscalar particle, the axion, would be desirable.

As the first bounds derived for point particles [52, 70] were least severe for pseudoscalars it was very tempting to suppose that the axion had been discovered at GSI [71]. For many people the axion became synonymous with the X particle although this possibility was ruled out pretty soon by more detailed theoretical calculations and additional experiments. Much effort was invested in constructing modified models, which the authors modestly called 'valid axion models' [72–74], which focused on the problem of finding a special combination of axion–quark couplings which would allow for an axion mass in the MeV region without being ruled out *a priori* by the negative outcome of previous axion search experiments. None of these attempts was able to explain the rather large production cross sections required by the GSI experiments and most never tried.

This whole discussion has turned out to be very important for the axion hypothesis itself. New varieties of axion models have been constructed, allowing for a greater variation in the properties of axions. These could play an important role if future experiments should again suggest the existence of new light particles.

The possibility should also be kept in mind that in a future theory the Higgs sector of the standard model might only be an effective description for complex extended states. In this case an analogue to the axion might exist which is no point particle.

An excellent review of the whole axion discussion can be found in [75]. We do not want to list again all the models and ideas which have been proposed in connection with the axion hypothesis. Instead we shall give an extremely simple description of the basic idea of these models and then concentrate on their phenomenological consequences.

The axion is tied to the problem of strong CP invariance, i.e. to the question why the QCD Lagrangian does not contain a term of the form

$$L=\frac{\alpha_{\text{strong}}\Theta}{8\pi}G^{a}_{\mu\nu}\tilde{G}^{a\mu\nu} \qquad \tilde{G}^{a\mu\nu}=\tfrac{1}{2}\varepsilon^{\mu\nu\rho\lambda}G^{a}_{\rho\lambda}. \tag{42}$$

It can be shown that any such term could be absorbed by a chiral transformation of quark fields

$$q_f(x)\rightarrow e^{i\alpha_f(x)\gamma_5}q_f(x) \qquad \sum_f \alpha_f=\tfrac{1}{2}\Theta. \tag{43}$$

The mass terms, which in the standard model are interpreted as Higgs–quark coupling terms, are, however, not invariant under this transformation. Its form is for the first generation of quarks

$$L=\frac{m_u}{v}(\bar{u}_R\bar{d}_R)\begin{pmatrix}-\varphi_0\\ \varphi_+\end{pmatrix}u_L-\frac{m_d}{v}(\bar{u}_R\bar{d}_R)\begin{pmatrix}\varphi_-\\ \varphi_0^*\end{pmatrix}d_L+\text{HC}. \tag{44}$$

The Higgs field has two components φ_0 and φ_+ and both of them can have different chiral transformation angles β_0 and β_+:

$$\varphi_0\rightarrow e^{i\beta_0}\varphi_0 \qquad \varphi_+\rightarrow e^{i\beta_+}\varphi_+. \tag{45}$$

Thus the Lagrangian is invariant under the chiral transformation (43) only if

$$\text{(i)}\ 2\alpha_u+\beta_0=0 \qquad \alpha_u+\alpha_d+\beta_+=0$$

$$\text{(ii)}\ \alpha_u+\alpha_d-\beta_+=0 \qquad 2\alpha_d-\beta_0=0$$

and thus

$$\alpha_d=-\alpha_u=\beta_0/2 \qquad \beta_+=0 \tag{46}$$

is fulfilled. This implies, however, that the sum of α_u and α_d vanishes and the Θ term persists (unless one of the quarks is massless and therefore would not couple to the Higgs fields).

The basic idea of the axion models is now simply to assume that the Higgs fields in the two terms of equation (44) are not just the charge conjugate of one another, but two completely different fields φ and φ' with different chiral transformation angles β_0, β_+, β_0', β_+'. Then α_u and α_d obviously can have arbitrary values without spoiling the chiral symmetry of the model:

$$L=\frac{m_u}{v}(\bar{u}_R\,\bar{d}_R)\begin{pmatrix}-\varphi_0\\ \varphi_+\end{pmatrix}u_L-\frac{m_d}{v}(\bar{u}_R\,\bar{d}_R)\begin{pmatrix}\varphi'_-\\ \varphi_0'^*\end{pmatrix}d_L+\text{HC}. \tag{47}$$

As a consequence of this enlargement of the Higgs sector one has now two vacuum expectation values v_1 and v_2 and an additional neutral and two additional charged Higgs fields. The charged components are assumed to be very heavy and thus unimportant and the two neutral components give the original Higgs field $\eta(x)$ and a new pseudoscalar field which is the axion $A(x)$:

$$\varphi(x)=\begin{pmatrix} 0 \\ v_1+\eta(x)+\mathrm{i}v_2 a(x) \\ \sqrt{v_1^2+v_2^2}+\mathrm{O}(a^2) \end{pmatrix}$$

$$\varphi'(x)=\begin{pmatrix} 0 \\ v_2+\eta'(x)+\mathrm{i}v_1 a(x) \\ \sqrt{v_1^2+v_2^2}+\mathrm{O}(a^2) \end{pmatrix}. \tag{48}$$

Equations (47) and (48) determine all the properties of the axion, e.g. the coupling to the up quark:

$$\frac{m_{\mathrm{u}}}{v_1}\bar{u}_{\mathrm{R}}\varphi_0 u_{\mathrm{L}}+\mathrm{HC}\rightarrow m_{\mathrm{u}}\bar{u}u+\frac{m_{\mathrm{u}}}{v_1}\bar{u}u\eta+\frac{v_2}{v_1}\frac{m_{\mathrm{u}}}{\sqrt{v_1^2+v_2^2}}\bar{u}\mathrm{i}\gamma_5 ua. \tag{49}$$

It turns out that all these couplings are determined by just one free parameter, namely the ratio of the two vacuum expectation value $x=v_2/v_1$.

The crucial point of the axion hypothesis is now that the Higgs field has to be minimised including the axion field and it was shown [7, 76] that this minimisation should lead to an axion component which corresponds to a chiral transformation cancelling any previously present Θ term.

All upper components of quark doublet (u, c, t, . . .) couple with x to the axion, whereas all lower components (d, s, b, . . .) couple with $1/x$. The original axion model [76] could therefore be excluded by two complementary experiments studying the decay of heavy mesons. The fact that no axion decay of the J/ψ particle

$$\mathrm{J}/\psi\equiv\bar{\mathrm{c}}\mathrm{c}\rightarrow\mathrm{a}+\gamma \tag{50}$$

has been observed leads to an upper bound for x, whereas the non-observation of the analogous reaction

$$\Upsilon\equiv\bar{\mathrm{b}}\mathrm{b}\rightarrow\mathrm{a}+\gamma \tag{51}$$

provides a lower bound for x. In other words the product of the respective branching ratios is independent of x and should be

$$\mathrm{BR}(\Upsilon(3S)\rightarrow\mathrm{a}+\gamma)\mathrm{BR}(\mathrm{J}/\psi\rightarrow\mathrm{a}+\gamma)=1.6\times10^{-8} \tag{52}$$

whereas earlier experiments [77, 78] found an upper bound of 6×10^{-10} for this product, thus excluding the existence of an axion. This argument is, however, only valid for a limited range of axion lifetimes. In particular the Υ decay experiments lost their sensitivity for a sufficiently fast axion decay. For an axion mass of 1.8 MeV this occurs for lifetimes shorter than 2×10^{-12} s, for more massive ones much earlier [79, 80]. Thus the J/ψ–Υ argument turned out to be invalid for the hypothetical X particle (as had the early nuclear decay searches, see §2.2). This led to a substantial amount of otpimism that the axion might indeed have been found, which, however, was soon refuted by new Υ-decay experiments [81, 82] and the new nuclear conversion experiments discussed in §2.2.

Meanwhile, however, it was noticed that the standard axion model does not use the most general form of the axion–quark coupling. As we have discussed, the chiral transformation of a single quark field can cancel any Θ term. Therefore it is not necessary to couple all quark families to the two Higgs doublets φ and φ'. If one uses, for example, the normal Higgs coupling equation (44) for the second doublet (c, s) the axion does not couple to the J/ψ and the above-mentioned argument collapses. This new class of models violates universality, which might or might not be an acceptable feature. It is obvious that such models cannot be ruled out conclusively, because the axion could, for example, couple exclusively to a heavier hitherto unknown quark generation. To be able to explain the GSI experiments they would, however, have to couple to the first generation of quarks and thus to protons and neutrons. These couplings were severely constrained by the nuclear decay experiments, which thus ruled out that a sufficient number of axions could be produced by nuclear bremsstrahlung or a similar process.

The coupling of the axion to the first-generation quarks is also strongly constrained by the fact that the decay $K^+ \to \pi^+ a$ was not observed. More precisely, its branching ratio has to be smaller than [80]

$$\mathrm{BR}(K^+ \to \pi^+ a) < 3.8 \times 10^{-8}. \tag{53}$$

The constraints on the axion couplings one can obtain from this bound are partially model dependent. The most direct bound is obtained for the flavour-changing coupling of the axion to the strange and down quark:

$$L = f_{ds} \bar{s} i \gamma_5 d a + \mathrm{HC}. \tag{54}$$

One gets [7]

$$|f_{ds}|^2 < 10^{-13}. \tag{55}$$

Higher-order processes allow also for kaon decay into a pion and an axion if f_{ds} is exactly zero. Their calculation is model dependent but the relevant coupling constants f_u and f_d can probably not be much larger than 10^{-5}.

Another constraint can be derived from the pion decay channel

$$\pi^+ \to a + e^+ + \nu_e \to e^+ + e^- + e^+ + \nu_e. \tag{56}$$

Krauss and Wise [83] calculated the branching ratio to be

$$\mathrm{BR}(\pi^+ \to a + e^+ + \nu_e) = 3 \times 10^{-6} \frac{m_u}{m_d} \left(\frac{\mathrm{m(axion)}}{1.8\ \mathrm{MeV}} \right)^2. \tag{57}$$

The experimental upper limit of 10^{-10} obtained by Eichler *et al* [84] again rules out the possibility that the X particle could be the axion (though the result [53] is somewhat model dependent).

In the course of the discussion of a possible connection between the GSI data and the axion hypothesis it was noted that the strong electromagnetic fields present in heavy ion collisions generate an effective space-dependent Θ term and thus might lead to non-perturbative processes like the formation of a local axion condensate:

$$\mathrm{constant} \times \boldsymbol{E} \cdot \boldsymbol{B} G^a_{\mu\nu} \tilde{G}^{a\mu\nu} = \Phi_{\mathrm{eff}}(x) G^a_{\mu\nu} \tilde{G}^{a\mu\nu}. \tag{58}$$

A more detailed study [65] showed that such a process was not likely to take place. This idea was, however, revived in connection with a speculation discussed in §6.3.

6. The situation for extended particles

Scenarios postulating new elementary pointlike particles are severely constrained by the bounds discussed so far. We therefore want to discuss next the possibility that the standard model as it is stands today, or with only minor adjustments, could allow for new complex bound states. If this were the case the basic equations and assumptions would be unaffected and all the successes of the standard model would be conserved. As our knowledge of the phenomenological consequences of the standard model is either based on perturbation theory (like QED) or is quite vague (as for the non-perturbative properties of QCD) it would not be such a great surprise if as yet unknown solutions existed.

In fact the existence of new complex bound states of known particles is far more probable than that of unknown elementary particles (with the exception of the axion) and we have to check how the bounds discussed so far change for such extended objects.

The first and perhaps most fundamental problem a theoretical analysis has to face for arbitrary composite objects is that their interactions and their equation of motion are in principle unknown. For pointlike particles there exist only very few renormalisable theories. As renormalisability can only be proven for spin-zero and spin-one theories there are only four cases which have to be considered (scalar, pseudoscalar, vector and axial-vector). Already for the spin-two case it is not clear whether a well defined renormalisable theory (which would describe, for example, gravitons) does exist. For extended objects renormalisability is not a valid criterion as it is dominated by the substructure of the X particle, not by its phenomenological properties at some finite (and in our case small) momentum transfer. For the same reason there is no clear constraint for its angular momentum. In principle the X could be a spin-20 object, coupling to one of the many posible rank-20 tensors which can be constructed from the momentum and spin vectors at one's disposal. Finally, many-body factors could play a role. An object which would only couple to, say, ten photons would be produced much more easily in strong electromagnetic fields.

All these caveats should be kept in mind during the following discussion, but they should not be taken too seriously. It is hard to believe that high-spin objects could exist which do not belong to a multiplet containing also spin-zero or spin-one states. (In addition one would expect similar particles belonging to low-spin multiplets.) These states should then all have comparable properties such that bounds derived for the low-spin states should be approximately true also for higher-spin states and for spin-zero and spin-one objects it should be possible to use the simple couplings of equation (6) and describe all effects due to the finite size by suitable phenomenological corrections like form factors.

We shall only demonstrate the effect of a finite size for one example, namely the bremsstrahlung experiments and their constraints for the X–electron coupling.

6.1. Bremsstrahlung of extended particles

Bremsstrahlung processes are usually calculated using the Weizäcker–Williams approximation. This approximation makes use of the fact that the Coulomb-field of a nucleus at rest can be substituted by a suitable combination of plane wave photons in the rest system of a fast moving electron.

As we are interested in the effects of a form factor we cannot use this approximation. Instead we calculated the two relevant bremsstrahlung graphs in full detail, see figure 20.

The form factor of the X particle G, coupling to an electron with incoming momentum p_i and outgoing momentum p_f can only depend on kinematic invariants.

As the X particle is supposedly on the mass shell $Q^2 = -q^2 = m_X^2$ we are left with only one such quantity, namely $\nu = p_i q$ (see figure 19). Either the incoming or the outgoing electron is on the mass shell which allows the expression of all other quantities in terms of $p_i q$. For $p_i^2 = m_e^2$ we have

$$p_f^2 = (p_i - q)^2 = -2p_i q + m_X^2 + m_e^2 \qquad p_f q = p_i q - m_X^2. \tag{59}$$

For corrections of up to 1 MeV^2 the only interesting invariant is therefore ν and we next have to find out how this can be combined with the X radius to give a meaningful dimensionless quantity. The expression we propose is $\nu R_X/M_X$ which, in the rest frame of the X particle, is just the ratio of its radius to the Compton wavelength of the incoming electron. As the choice of ν was only unique up to corrections of the order of 1 MeV^2 the form factor we use, and thus our calculation, becomes unreliable for $R_X > 200$ fm.

The usual electromagnetic form factor of, for example, the proton depends only on $Q^2 R_P^2$, i.e. the coupling is only reduced for off mass shell photons and this reduction depends on the proton radius. In our problem the X particle which corresponds to the photon is on mass shell and we are interested in effects due to its radius, whereas photons have no intrinsic size.

The situation we are interested in is thus not comparable with those usually faced. The only parallel is the pion–nucleon vertex which should show both a Q^2 dependence due to the nucleon radius and a ν dependence due to the finite pion size. As the pion is small compared with the nucleon and also strongly interacting the situation is rather unclear in this case. Experimentally a ν dependence has not so far been established.

In contrast to the pion–nucleon coupling we are facing a situation in which the emitting electron is point like while the emitted X particle could be extremely large.

Although a ν dependence has not been found so far in any experiment we believe that it should exist. If it did not exist, then the beam-dump results would exclude any particle interpretation of the GSI data, whether it assumes point like or extended particles. We shall show that even if it existed the beam-dump experiments still give very strong bounds unless R_X is much larger than 100 fm. For such large radii our calculation becomes unreliable anyway, not only due to the reason discussed above, but also because the photons in figure 20 which have typically 1 MeV momentum start to resolve internal structures of the X particle, such that the two vertices in figure 20 start to overlap.

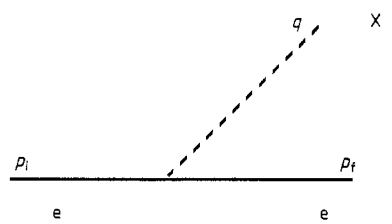

Figure 19. The X–photon vertex.

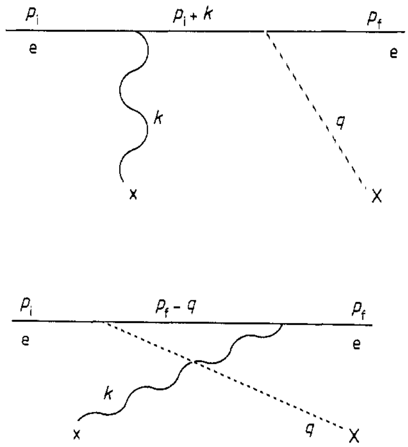

Figure 20. The two graphs contributing to the X bremsstrahlung.

In our calculation [85] two extreme choices for the form factor were made, a very slow, monopole-like decrease

$$G(\text{monopole}) = \left[1 + \left(\frac{p_i q}{m_X} R_X\right)^2\right]^{-1} \tag{60}$$

and a very steep, step-function-like decrease

$$G(\text{theta}) = \Theta\left(\frac{m_X}{R_X} - p_i q\right). \tag{61}$$

The contributions from the two graphs in figure 20 were calculated for the scalar, pseudoscalar, vector and axial-vectorial case. The results for a scalar X are nearly the same as for a pseudoscalar X and those for a vector are nearly equivalent to the axial-vectorial ones. We checked our results by comparisons with earlier calculations done in the Weizäcker–Williams approximation [86] for point particles.

Figure 21 shows the dependence of the cross section on the X radius for the pseudoscalar and vector case and for a kinematic situation typical for the SLAC beam-dump experiment [23]. x is the fraction of the momentum of the incoming electron carried by the X particle and $F(x)$ is defined by

$$\frac{d\sigma}{dx} = \frac{2(Z\alpha)^2\alpha_X}{m^2} F(x). \tag{62}$$

The most striking properties of the curves in figure 21 are that the cross section is reduced substantially only for very large values of R_X, although the electron momentum is 9 GeV and that the cross section actually increases for moderate values of R_X.

The first property is due to the fact that the bremsstrahlung cross section is very strongly forward peaked, such that the four-vector q_μ is nearly colinear to the four-vector $p_{i\mu}$. Therefore ν is very small compared with the naive expectation $p_i q = O(\text{MeV}^2)$ and extremely large values of R_X are required to get a sizeable suppression.

The rise of the cross section for small values of R_X is due to the fact that for point particles there is a very strong cancellation between the two graphs in figure 20. As the form factors for both graphs are different, this cancellation becomes less and less effective as R_X increases, leading to a rise in the cross section.

From these results it seems that the bounds from the beam-dump experiments are actually more severe for particles with a radius of a few fermi than for point particles. Radii of at least 100 fm are needed to weaken them to such an extent that they become compatible with the particle interpretation of the GSI lines.

The only alternative possibility to avoid the constraints from the beam-dump experiments would be to assume an extremely large X–nucleon cross section such that any X particle produced would be absorbed in the beam-dump. The required cross sections are, however, much too large (≥ 50 mb [23]) to be compatible with ordinary nuclear physics.

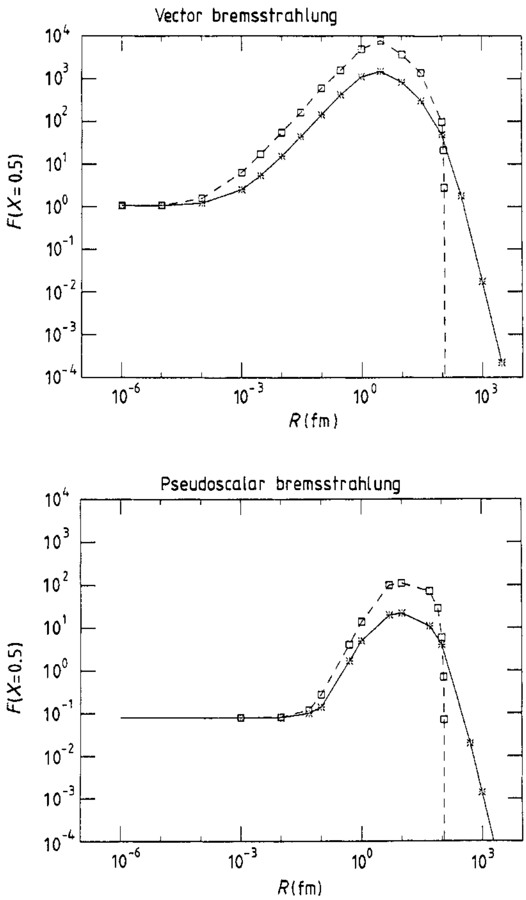

Figure 21. The effect of the form factor for pseudoscalar and vector particles. The full curve was obtained with the theta-function form factor and the broken curve with the monopole-like form factor.

Large X particle sizes are also suggested by the energy difference between the different lines of roughly 200 keV, the inverse of which might give some hint as to the spatial extension of the object having this spectrum.

Many of the bounds derived in § 3 would become invalid or easily explained if the X radius were really that large. The coupling to nuclei would be suppressed roughly speaking by the ratio of the volumes and the bounds on the X–photon coupling by still higher powers of R_{nucleus}/R_X, depending on the multipolarity.

6.2. *Bag models for extended particles*

The simplest way to construct a model for an extended X particle is to copy the features of the MIT bag model. As the particles in the bag are free the X radius and mass are related by the uncertainty relation, implying X radii of several hundred fm. As a consequence of this large size the subparticles (corresponding to the quarks) can hardly be charged as the large polarisability of the X-bag would otherwise result in unacceptable corrections to vacuum polarisation in, say, the hydrogen atom. At present it is not clear whether such models are in contradiction with, for example, the Lamb shift experiments or whether their contribution can be reduced by the introduction of a suitable rest interaction between the subparticles.

Schramm *et al* [26] emphasised this problem but also argued that the polarisation of such bags in the electric field of two heavy nuclei could drastically reduce their energy, even down to negative values, and that therefore such objects could be produced spontaneously in heavy ion collisions.

Wong [87] formulated a bag model for electrons and positrons, based on the idea of a possible change in the QED vacuum structure which we shall discuss in § 6.3. He started from the usual expression for the bag energy:

$$E(R)=\frac{4\pi}{3}R^3B+\frac{\Sigma_i \varepsilon_i-\varepsilon_{\text{cm}}}{R} \tag{63}$$

where B is a suitably chosen bag constant and ε_i are the one particle energies of electrons and positrons confined within the bag. He chose $B^{1/4}=0.25$ MeV which led to a ground state radius of 510 fm. He introduced spin–spin and spin–orbit interactions and was thus able to fit some of the different sum energies claimed so far by the positron experiments. The problem of this approach clearly is to justify the bag model description. The X particle has to propagate into a field-free region before decaying as otherwise electrons and positrons would have different energies. Thus the basic assumption of the bag model, namely the existence of a complex vacuum state in which the confined particles cannot propagate is completely unrealistic.

Schramm *et al* [26] proposed a bag model containing a completely new class of light electrically charged fermions. They also used the standard MIT expression for the energy, equation (63), and introduced a spin–spin interaction to obtain a suitable mass spectrum. The radius of their bags is $\geqslant 1000$ fm. The principle problem of this approach besides the constraints from vacuum polarisation is that it leaves the framework of the standard model and introduces a whole new family of fermions and a new force acting between them just to describe a few electron–positron correlations of which it is still unknown whether they are due to a two-body decay or not.

Similar bag models can also be motivated along the lines of Celenza *et al* [88] who argued that the X particle might be a soliton produced in a locally modified QED phase, or following the idea of Shaw [89] who related them to a specific model for a spontaneously broken QCD.

All such attempts are able to give a good description of the 'observed' mass spectrum, which probably means that just three lines with very similar energy do not specify the interaction which generates them. The main point common to all these models is that the objects in question would have to have a radius of at least several hundred fermi.

A series of authors have discussed the possible existence of very small magnetically bound positronium resonances [90, 91] with a radius of only a few fm. We do not want to discuss these hypotheses in this review as they are not only inconsistent [92–94] but also ruled out experimentally by the beam-dump experiments we discussed in § 2.2 and 6.1.

6.3. Localised vacuum excitations

We have shown above that the GSI coincidences can hardly be explained by the decay of a new particle. The collection of constraints which has been derived so far limit this hypothesis to very exotic objects. The only acceptable X particle would have to be extremely large, namely at least 100 fm possibly 1000 fm, and its constituents very probably cannot be charged.

Furthermore we have argued that these objects should be accommodated within the standard model. To go beyond the standard model postulating completely new particle families or interactions is clearly not justified by the scarce experimental data. Such a step should only be a last resort if new experiments would (unexpectedly) find some undisputable evidence for the X particle.

Any extended object of the kind just described can be thought of as a localised vacuum excitation. As the standard model includes three different vacua, namely the QED-vacuum, the Higgs vacuum, and the QCD-vacuum, all speculations proposed so far can be classified in terms of the states affected.

The intrinsic energy scale of the X particle is 1 MeV. QED has about the same energy scale $m_e = 0.511$ MeV, while the QCD-scale is 150 MeV and the Higgs energy scale 170 GeV. Thus as far as the energy scale is concerned QED seems to be the most promising candidate. Unfortunately its vacuum state is trivial (under normal circumstances) allowing for no interesting non-perturbative complex states. It was suggested by Celenza *et al* [95], Caldi and Chodos [96] and by Ng and Kikuchi [97] that the presence of a strong electromagnetic field might completely alter the properties of QED. This extreme hypothesis is motivated but not justified by the fact that QED shows a phase transition into a confining phase if the fine-structure constant is made large enough, i.e. of order unity (see references in [96]). This hypothesis was then used to motivate simple bag-like models like those discussed in § 6.2.

The assumption that strong electromagnetic fields could reduce the critical fine-structure constant for the QED phase transition to less than 1/137 is, however, in no way plausible. A QED phase transition would most probably have to be triggered by a condensation of e^+–e^- pairs. As the dominant electric fields present in heavy-ion collisions tend to separate electrons and positrons they should rather hinder than provoke such a condensation. A similar conclusion was reached by Peccei *et al* [98].

Another point of criticism is that the X particle would have to decay into e^+–e^- pairs far away from the nuclei to explain why electron and positron have nearly the same energy. This implies that a region of modified QED vacuum would have to be somehow ejected from the combined nuclear system and would have to be sufficiently stable to survive at least 10^{-18} s. As long as no mechanism is proposed which could result in such a behaviour the relevance of these speculations for the GSI experiments is unclear.

Although these simple arguments might sound convincing they are no proof that a QED phase transition cannot be induced in the heavy-ion collisions under investigation. The only clear statement in this matter can be expected from serious calculations using non-perturbative techniques. The first step in this direction are lattice-gauge calculations by Dagotto, Kogut, Kocic and Wyld [99, 100]. Because the correct treatment of light fermions, namely the electrons, is so crucial for this problem these calculations are plagued by severe theoretical uncertainties. Furthermore only small lattices can be studied numerically such that all phenomena with a length scale substantially different from the Compton wavelength of the electron cannot be investigated.

The discretised action used in [100] is

$$S = S_0 + \sum_{x,y} \bar{\psi}_x M_{x,y} \psi_y$$

with

$$M_{x,y} = \frac{1}{2} \sum_{\mu} \eta_{x,\mu} [\exp(f(\boldsymbol{x})\delta_{\mu,4}) U_{x,\mu} \delta_{y,x+\mu} - \exp(-f(\boldsymbol{x})\delta_{\mu,4}) U^*_{y,\mu} \delta_{y,x-\mu}] + m\delta_{x,y}$$

and

$$S_0 = \beta \sum_{p} Re(U_p) \tag{64}$$

where p denotes the plaquettes and μ the links of a four-dimensional cubic lattice. $f(\boldsymbol{x})$ is chosen to be Z/r to describe QED in a Coulomb field. (Please note that Z therefore corresponds to $Z\alpha$ in the usual notation.) The electrons are treated as staggered fermions. As usual β has the meaning of an inverse coupling constant. So, large values of β correspond to weak coupling and small values of β to strong coupling. Figure 22 shows the schematic phase diagram suggested by the results of this calculation. Three phases are visible. Phase II is the usual free QED vacuum state, without any condensate and with no net fermion number. Phase III is the well known charged vacuum phase the search for which motivated all of the GSI experiments [9]. Here one has a net electron number $n_f \neq 0$ but no condensate. Phase I is the new confining vacuum phase which for $Z = 0$ is only reached for $\alpha > \alpha_c$, corresponding to $\beta < \beta_c$. The crucial question is now whether the right boundary of phase I bends to the right for large Z as postulated in [95–97]. This is obviously not the case.

While the question of the QED vacuum structure under extreme conditions is definitely a fascinating one which should and will be studied further, so far it does not seem to have any relevance for the interpretation of the GSI data.

For the QCD vacuum the mismatch in energy scale is large (the ratio being of the order 100) but perhaps not insurmountable. Schäfer *et al* [101] proposed that the

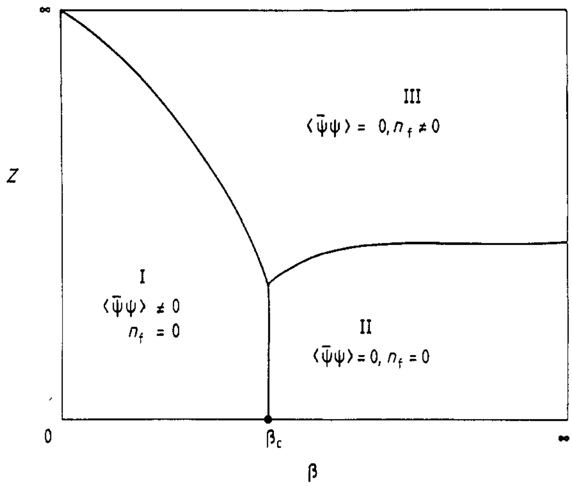

Figure 22. Schematic phase diagram of QED in an external Coulomb field. Both the electron condensate $\langle\psi\psi\rangle$ and the electron density n_f are measured close to the electric charge.

strong electromagnetic fields acting as an effective space-dependent Θ term (see the last paragraph in §5) might induce a pseudoscalar gluon condensate $\langle G^{\mu\nu}\tilde{G}_{\mu\nu}\rangle$. As a space-dependent Θ term couples through the triangle anomaly to the gluon propagator the effective scale parameter for such excitations could be different from the usual Λ_{QCD}. (Λ_{QCD} depends strongly on the renormalisation properties which would be modified by the Θ term.) Details of this idea can be found in [101] and [6]. We do not want to discuss this idea any further at this point as no serious calculations have been done so far to corroborate this hypothesis or to rule it out. Without such a calculation this speculation, as the one presented before for the QED vacuum, is pure fantasy.

The first scenario suggesting a localised complex vacuum change was actually the first paper which tried to interpret the Z-independent positron lines as due to a particle decay [4]. In this paper non-trivial charged excitations of the Higgs-vacuum were postulated decaying into a positron–neutrino pair. The subsequent coincidence experiments of the EPOS group showed instead that one would have to search for a neutral object decaying into an electron–positron pair and furthermore a more detailed analysis showed that the proposed mechanism would require a completely unrealistic electric field strength [5]. Recently a similar idea was proposed by Celenza *et al* [102].

It is obvious that all of these speculations, while being interesting in their own right are very far from being of relevance for the interpretation of the GSI data. They show that the low energy properties of the standard model are not understood as completely as one might expect.

7. Conclusions

In this review we have tried to give an overview over the results obtained in the last years in the search for new light particles (with spin one or zero). Most of these experiments were done to prove or disprove the hypothesis that the coincident electron–positron lines observed at GSI are due to the decay of such a particle. We have tried to present these results in as general a manner as possible, stressing all those points which are of importance for every hypothesis postulating such particles, and leaving out most of the calculations which have only been relevant for the GSI experiments and are outdated now by recent developments.

In § 2 we have reviewed the GSI experiments concluding that while the existence of the correlated electron and positron peaks is well established it is not clear whether their characteristics are in agreement with a two-body decay of an unknown particle.

We have also discussed some other particle speculations to show that there is a general demand for the comprehensive investigation done in connection with the GSI peaks.

Complementary experiments searching for light particles which are also reviewed in § 2 have provided severe constraints. Most important are the beam-dump experiments which rule out that any particle with radius smaller than 100 fm can be responsible for the GSI coincidences.

In § 3 we have presented the most crucial bounds for the coupling constants of hypothetical particles which are obtained from the analysis of high precision atomic and nuclear experiments. These bounds are the most important theoretical results and their relevance does not depend on any specific particle hypothesis.

For the GSI events these bounds imply that a highly non-perturbative production mechanism is needed to explain the observed cross sections and that the hypothetical X particle has to be extended.

We also gave a very short review of the new axion models the development of which was spurred by the GSI experiments, as well as an overview of possibilities for complex low-energy vacuum excitations in the standard model.

Our general conclusion is that many interesting questions have been raised promising a very active development of this field of research over the next few years.

With respect to the question whether the GSI data signal the existence of a new particle we must conclude that the doubts raised by the experiments are much aggravated by careful theoretical analyses. A final word in this matter can, however, only come from improved experiments.

To many people in the field, like Greiner [103], it seems increasingly probable that the coincident electron and positron lines are caused by a combination of some exotic conversion process and complex atomic physics phenomena like the one proposed by deReus *et al* [104].

Acknowledgments

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